Turbulence Analysis Of An Experimental Flux-Rope Plasma

D. A. Schaffner
Vyacheslav S. Lukin , '00
A. Wan
Michael R. Brown
Swarthmore College, doc@swarthmore.edu

Recommended Citation
https://works.swarthmore.edu/fac-physics/122

This Article is brought to you for free and open access by the Physics & Astronomy at Works. It has been accepted for inclusion in Physics & Astronomy Faculty Works by an authorized administrator of Works. For more information, please contact myworks@swarthmore.edu.
Turbulence analysis of an experimental flux-rope plasma

This content has been downloaded from IOPscience. Please scroll down to see the full text.

(http://iopscience.iop.org/0741-3335/56/6/064003)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 130.58.65.20
This content was downloaded on 09/02/2015 at 21:05

Please note that terms and conditions apply.
Turbulence analysis of an experimental flux-rope plasma

D A Schaffner¹, V S Lukin², A Wan¹ and M R Brown¹

¹ Swarthmore College, Swarthmore, PA, USA
² Space Science Division, Naval Research Laboratory, Washington, DC, USA

Received 8 October 2013, revised 16 December 2013
Accepted for publication 30 December 2013
Published 12 February 2014

Abstract
We have previously generated elongated Taylor double-helix flux-rope plasmas in the SSX MHD wind tunnel. These plasmas are remarkable in their rapid relaxation (about one Alfvén time) and their description by simple analytical Taylor force-free theory despite their high plasma $\beta$ and high internal flow speeds. We report on the turbulent features observed in these plasmas including frequency spectra, autocorrelation function, and probability distribution functions of increments. We discuss here the possibility that the turbulence facilitating access to the final state supports coherent structures and intermittency revealed by non-Gaussian signatures in the statistics. Comparisons to a Hall-MHD simulation of the SSX MHD wind tunnel show similarity in several statistical measures.

Keywords: turbulence, flux-rope, solar wind, simulation, MHD

Online supplementary data available from stacks.iop.org/PPCF/56/064003/mmedia

1. Overview

Flux ropes observed in the heliosphere have two striking properties. First is their rapid emergence. Whether in the magnetosphere or in the solar corona, these large scale structures emerge rapidly, often in just a few Alfvén crossing times of the system. Second is their long lifetimes. Once formed, these structures persist for long times despite being embedded in turbulent magnetohydrodynamic (MHD) plasma.

Flux ropes have recently been observed in situ at the subsolar magnetopause [1]. Since magnetospheric flux ropes evolve rapidly, observations of these flux ropes tend to be made at later stages of their evolution. However, in this remarkable coordinated observation using three THEMIS spacecraft, a flux rope is caught in the process of forming, revealing properties that are fundamentally three-dimensional (3D).

Flux ropes are also observed remotely in the solar atmosphere [2]. On July 19, 2012, an eruption occurred on the solar surface producing dynamical magnetic activity resulting in a destabilized flux rope, the acceleration of a fast ($\approx 1000 \text{ km s}^{-1}$) coronal mass ejection, and a long-lived solar coronal arcade. The long-lived structure is remarkable in its nearly semi-circular geometry, and the persistence of a ‘coronal rain’ from the loop tops for nearly 24 h. The observation was made with the Solar Dynamics Observatory’s AIA instrument on the sun’s lower right hand limb. This represents the first direct evidence of a fast CME driven by the prior formation and destabilization of a coronal magnetic flux rope.

In a laboratory plasma, Gray et al [3] recently reported on the observation of a long-lived helical flux rope called a Taylor double-helix in the SSX MHD wind tunnel. The Taylor double-helix is the natural relaxed state of MHD plasma confined in a long, perfectly conducting cylinder [4]. In the case of an infinite cylinder of radius $a$, the minimum energy state possesses $ka = 1.292$, which is within 5% of the infinite cylinder’s $ka = 1.234$.

Servidio et al [6, 7] detail simulations which observe the rapid and simultaneous magnetohydrodynamic relaxation into...
localized patches of plasma with near alignment of $\mathbf{B}$ and $\mathbf{J}$. These patches of locally relaxed plasma can then negotiate with adjacent patches to reach a globally relaxed state on a longer time scale. However, many of the characteristics of the relaxed state will be evident locally. This localized relaxation might explain the rapidity of the transition observed in the double-helix plasmas. A fully relaxed Taylor state would be expected to have a flat lambda profile (where $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ governs the equilibrium). The reported lack of a flat radial lambda profile could also be a consequence of a patchy relaxation.

We consider the possibility that the MHD turbulent flow in the SSX wind tunnel contains patches of locally relaxed plasma with reconnection sites at the boundaries. A fully relaxed flow might be expected to exhibit Gaussian statistics in its fluctuations and power-law behavior for the power spectra; conversely, a flow containing coherent structures and reconnection sites should exhibit non-Gaussian statistics. Simulations show that coherent structures appear rapidly, in less than one dynamical time. Large numbers of reconnection sites can be identified statistically in MHD turbulence studies [8, 9]. A statistical way to find these coherent structures is to identify rapid changes in the magnetic field vector. A useful technique is to generate a probability distribution function (PDF) of vector increments [10, 11].

Greco, et al [10] identified intermittent structures in MHD turbulence simulations using statistical techniques, then connected the structures with regions of high power density. Using data from a high resolution 3D MHD simulation, PDFs of magnetic field vector increments $\Delta \mathbf{B} = |\mathbf{B}(s+\Delta s) - \mathbf{B}(s)|$ were constructed for two scale separations $\Delta s$; one much smaller than the correlation scale $\lambda_c$ and another $\approx 2\lambda_c$. It was found that the PDF of $\Delta \mathbf{B}$ at the larger increment was close to Gaussian indicating that increments larger than the correlation scale are normally distributed. However, the PDF at the smaller increment had substantial tails in the distribution and furthermore, had the same distribution as the PDF of a component of the current density, indicating that the non-Gaussian statistics were correlated with intense current sheets. The identification of discontinuities using this spatial interval method was also compared with coherent structures identified using intermittency statistics and found the techniques to be similar [10].

In a follow-up investigation, Greco et al [11] studied the statistics of ACE solar wind data as well as 2D and 3D simulation data using the same techniques. Time series were analyzed for the solar wind data while spatial separations were analyzed from the simulations. First, it was found that the ACE solar wind data had nearly identical increment PDFs to the 2D simulation data. Second, specific features in the 2D simulation were shown to be correlated to departures from Gaussian statistics in the PDF: (a) a narrow inner peak is super-Gaussian and corresponds to small fluctuations in the lanes between magnetic islands; (b) an intermediate range is sub-Gaussian and corresponds to fluctuations in current cores inside magnetic flux tubes; (c) at several standard deviations, there are super-Gaussian wings corresponding to coherent small-scale current-sheet-like structures that form the sharp boundaries between the magnetic flux tubes.

Non-Gaussian statistics and characteristic coherent structures are initiated almost identically in weakly dissipative and ideal systems [12]. Therefore we postulate that the origins of coherence and intermittency are essentially ideal, with dissipation acting only to limit growth of the smallest scale structures. We expect that the modest value of SSX MHD wind-tunnel Lundquist number ($S \sim 1000$) should not impact the emergence of coherent structures.

In this manuscript, we focus on the post-selective-decay phase of the double-helix, in particular, processes that rapidly evolve the state such as patchiness and the evolution of coherent structures. First, we report on the turbulent temporal power spectrum for various aspects of these experimental flux ropes and fit to power laws in order to compare to similar measurements in both the solar wind and MHD simulations. Then, we present MHD turbulence statistics that suggest the emergence of non-Gaussian structures. These experimental turbulent features are compared to synthetic data extracted from simulations of an MHD wind tunnel designed to have a geometry and plasma parameters matched to the SSX experiment. The simulations use both resistive-MHD and Hall-MHD numerical models within the HiFi framework [13].

2. Experiment

The flux ropes under investigation are formed in a wind-tunnel configuration of the Swarthmore Spheromak Experiment. A copper cylindrical flux conserver serves as the tunnel capped by two plasma gun electrodes whose extent limits the length of the tunnel to 86 cm as can be seen in figure 1. The radius, $R$, of the cylinder is 7.75 cm making the aspect ratio of the configuration, $L/R = 11.1$. Though slightly shorter than the tunnel reported in previous work [3], the aspect ratio is considered still large enough for comparison to an infinite cylinder in Taylor relaxation theory. The plasma itself is formed by the discharge of a 1 mF, 3.5 kV capacitor across a few centimeters wide gap between the tungsten-coated gun inner electrode and the outer wall into auffed volume of hydrogen gas. After ionization, currents of over 80 kA across the gap push plasma into the main section of the flux conserver through $J \times B$ forces. Magnetic coils coaxial to the gun electrode and flux conserver contribute the stuffing flux which allows for the formation of a spheromak at the gun edge. Given the high aspect ratio, the spheromak tilts, eventually forming a twisted double-helix Taylor state; this sequence has been shown to occur in a very short time span [3]. For this discharge configuration, the amount of current needed to push spheromaks off of the gun and into the main chamber is maintained for approximately 30 $\mu$s ($\sim 20–50 \mu$s from initial discharge). The discharge likely begins with the formation of a spheromak structure into the chamber vacuum, which may or may not completely break-off from the gun source before more plasma is introduced behind it. Thus these shots generally consist of a sustained injection of structured plasma rather than a single well-defined spheromak—a configuration called slow formation [14]. As this paper will discuss, the turbulence nature of this plasma is unaffected by the source physics as the selective decay of the plasma likely occurs locally.
Magnetic fluctuations are measured using an arrayed $\vec{B}$ probe; $\vec{B}$ in three directions $(r, \theta, z)$ is measured using three orthogonally oriented 0.3 cm single-loop coils at 16 locations separated by 0.4 cm, beginning 1 cm from the cylindrical axis. Signals are acquired using a DTaq digitizer at 14-bit resolution and 65 MHz sampling rate. Magnetic field vectors are computed through numerical integration of the $\vec{B}$ signal. Since magnetic fluctuations are observed over a wide range of frequencies, the frequency response of the probe and electronics have been calibrated to a pulsed current driver. Calibrated power spectra show some drop in power as a function of frequency, but this droop is typically an order of magnitude less than the drop seen in the data. Moreover, using a single-loop coil helps limit inductive effects which could limit bandwidth. Mach number fluctuations, as a proxy for velocity fluctuations, are measured using a Mach probe oriented along the $z$-axis [15] and located 1.5 cm from the inner edge of the flux conservor. Density measurements are made using a He–Ne (633 nm) laser interferometer through a diameter of the flux conservor set an axial distance of 21.5 cm from the midplane in the direction of the plasma source. In the data presented here, it is assumed that the plasma has already undergone initial Taylor relaxation by the time it reaches the midplane where most of the diagnostics are located. The chamber is pumped down to $\sim 6 \times 10^{-8}$ Torr and undergoes a helium glow discharge between runs, so it is assumed that any influence of impurities on the plasma is low.

An additional important point regarding the SSX MHD wind-tunnel experiment is that due to the nature of the plasma production method, the plasma does not have a dominant guide field direction; that is, the turbulence tends to be isotropic. This is evident in the similarity of the spectra for the three different probe directions, as discussed below.

3. HiFi simulation

Simulations of the SSX MHD wind-tunnel experiment have been performed with the HiFi spectral-element multi-fluid modeling framework [16, 17]. Within HiFi, the following set of normalized compressible Hall-MHD equations with wall recycling is advanced in time:

$$\frac{\partial \ln(\rho)}{\partial t} + \frac{1}{\rho} \nabla \cdot \left[ \rho \mathbf{v}_i - D_\rho \rho \nabla \ln(\rho) \right] = 0,$$  \hspace{0.5cm} (1)

$$\frac{\partial (\rho \mathbf{v}_i)}{\partial t} + \nabla \cdot \left[ \rho \mathbf{v}_i \mathbf{v}_i + (p_i + p_e) \mathbf{I} - \mathbf{\Pi}_i - \mathbf{\Pi}_e - D_\rho \nabla (\rho \mathbf{v}_i) \right] = \mathbf{J} \times \mathbf{B} - R_w(r) \rho \mathbf{v}_i,$$  \hspace{0.5cm} (2)

$$\frac{\rho}{\gamma - 1} \frac{\partial \mathbf{T}}{\partial t} + \rho \nabla \cdot \left[ \frac{1}{2} (\mathbf{v}_i + \mathbf{v}_e) \mathbf{T} \right] - \nabla \cdot q = \frac{1}{\rho} \nabla \cdot \left[ \rho D_\rho \nabla \mathbf{T} \right] - \frac{1}{\gamma - 1} \left[ \gamma - 2 \right] \left( p_i + p_e \right) \nabla T - R_w(r) \rho T + \frac{1}{2} \left[ (\mathbf{\Pi}_i + D_\rho \rho \nabla \mathbf{v}_i) \cdot \nabla \mathbf{v}_i + (\mathbf{\Pi}_e \cdot \nabla) \mathbf{v}_e + \eta |\mathbf{J}|^2 \right],$$  \hspace{0.5cm} (3)

where

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \mathbf{v}_c = \mathbf{v}_i - \mathbf{d}_i/\rho \mathbf{J},$$

$p_i = p_e = \rho T$,

$\rho$ is the plasma number density, $\mathbf{v}_i$ and $\mathbf{v}_e$ are the ion and electron velocity fields, $\mathbf{A}$ is the electro-magnetic vector potential, $\mathbf{B}$ is the magnetic field, $\mathbf{J}$ is the current density, $p_i$ and $p_e$ are the ion and electron pressures which are set equal to each other under the assumption of fast electron–ion temperature equilibration, $T$ is the temperature of either species, $\mathbf{\Pi}_i = \mu_i \nabla \mathbf{v}_i$ is the ion viscosity tensor, $\mathbf{\Pi}_e = \mu_e \rho \nabla \mathbf{v}_e$ is the anomalous electron viscosity tensor, $q = \kappa \nabla T$ is the thermal conduction heat flux, and $\gamma = 5/3$ is the ideal gas adiabatic index. These equations have been normalized in the standard fashion [13] by choosing three unit normalization quantities, which are set to be the unit length $R_0 = R = 7.8$ cm, the unit magnetic field strength $B_0 = 5$ kG, and the unit...
number density $n_0 = 10^{16}$ per cubic cm, which also sets the normalized ion inertial length coefficient $d_i = (c/\omega_{pi})/R_0 = 2.9 \times 10^{-2}$. This value for $d_i$ is used in the Hall-MHD simulations, while in the resistive-MHD simulations all two-fluid terms are neglected by setting $d_i = 0$. Note that, as described and shown below, the average electron number density in the simulations is $\sim 0.1n_0$, and therefore the effective normalized ion skin depth is $\sim 2.9 \times 10^{-2}/\sqrt{0.1} \sim 0.1$. Also, although experimental measurements of the plasma typically show a difference in ion and electron temperature, the two temperatures are set equal in the simulation as a simplifying assumption.

The resistivity $\eta = \eta(\rho, T, |J|)$ used in the simulations is a combination of the classical Spitzer resistivity and the semi-empirical Chodura resistivity designed to capture anomalous electron drag at low plasma density and high current density [18–20]:

$$\eta(\rho, T, |J|) = \frac{\eta_0^{Sp}}{T^{3/2}} + \frac{\eta_0^{Ch}}{\sqrt{\rho}} \left[1 - \exp\left(-\frac{\eta_0^{Ch} |J|}{3 \rho \sqrt{\gamma f}}\right)\right].$$

with the normalization constants $\eta_0^{Sp} = 10^{-4} \Lambda^{7/2} m_0 \mu_0 \varepsilon_0^{3/2}$, $\eta_0^{Ch} = 2.7 \times 10^{-4}$ for the Spitzer resistivity, $\eta_0^{Ch} = (0.1m_e/e\tau_{0}(\mu_0)/(1/L_0)) = 4.4 \times 10^{-3}$ and $\eta_0^{Ch} = (\sqrt{m_e/\tau_{0}(\mu_0)})(1/R_0 \tau_{0}(\mu_0)) = 2.9 \times 10^{-2}$ for the Chodura resistivity, expressed in the standard notation and with the Coulomb logarithm set to $\Lambda = 10$.

The wall recycling terms proportional to $R_w(r)$ in equations (2) and (4) represent the loss of plasma momentum and temperature to the wall under the assumption of near-wall particles impacting the wall at some velocity but sputtering off the wall with negligible velocity, followed by instantaneous thermalization. It is further assumed that all particles impacting the wall return into the plasma, an assumption that may be relaxed in future work. For the simulations described here, $R_w(r)$ is given by

$$R_w(r, z) = \frac{1}{\tau_w} \exp\left[-\frac{z}{\lambda_w}\right] + \exp\left[-\frac{L/R_0 - z}{\lambda_w}\right] + \exp\left[-\frac{1 - r}{\lambda_w}\right].$$

where $\lambda_w = 0.05\,\text{cm}$, equivalent to 0.39 cm, is the characteristic thickness of the near-wall recycling layer in the cylindrical simulation domain given by $(r, z) \in [0, 1] \times [0, L/R_0]$ with $L/R_0 = 11.1$. Due to lack of a good theoretical estimate for the characteristic wall recycling time $\tau_w$ in the SSX experiment, a series of simulations systematically varying $\tau_w$ was conducted to empirically determine its value by comparing the magnetic field decay times in the simulations to those determined from the experimental magnetic field measurements discussed below. (Note that since the wall recycling directly impacts the temperature of the plasma, it also significantly impacts the magnetic field resistive decay time via the temperature dependence of the Spitzer resistivity.) As a result of this single-parameter study with all other input parameters held fixed, the wall recycling time has been set to $\tau_w = 2\,\mu\text{s}$, equivalent to 1.4 $\mu$s.

Equations (1)–(4) also contain several terms proportional to a density diffusion coefficient $D_\rho$. The specific form for these has been derived by taking velocity moments of a modified Boltzmann transport equation for the ion distribution function $f_i(t, r, v)$ [21]; with the Boltzmann equation modified by adding an extra $D_\rho \nabla^2 f_i$ diffusion term designed to represent sub-grid scale plasma mixing.

The simulations are performed on a cylindrical mesh with the spectral elements uniformly distributed in each direction with a grid of $(n_r, n_\theta, n_z) = (24, 24, 180)$ elements and the 3rd order modified Jacobi polynomials used to expand the solution in each direction within each element [17]. Thus, the simulations’ effective mesh size is $(N_r, N_\theta, N_z) = (72, 72, 540)$, providing radial resolution of 0.11 cm, angular resolution of 5°, and axial resolution of 0.16 cm.

The simulations are initialized with the plasma at rest but far from an equilibrium, with nearly all of the plasma density, thermal energy, and magnetic field concentrated in one end of the tunnel, within $0 < z < 1.5$. This is intended to mimic the initial formation and the rapid expansion of the magnetized plasma cloud into the near-vacuum chamber in the SSX experiment. Specifically, the plasma number density is initialized with the following spatial distribution:

$$\rho(z)_{|z=0} = \begin{cases} 1, & 0 \leq z < (1 - \lambda_p), \\ \frac{1 + \rho_{min} - 1 - \rho_{min}}{2 - \sin\left(\frac{\lambda_p}{2}\right)}, & (1 - \lambda_p) \leq z \leq (1 + \lambda_p), \\ \rho_{min}. & (1 + \lambda_p) < z \leq 11.1, \end{cases}$$

where $\lambda_p = 0.4$ is the half-width of the transition from the high-density region to the much lower density background tunnel plasma at $\rho_{min} = 10^{-2}$. The initial temperature distribution assumes an adiabatically expanded plasma with

$$T(z)_{|z=0} = T_0 \rho^{-\gamma - 1}(z)_{|z=0}$$

and $T_0 = 0.1$, corresponding to the temperature of 6.2 eV in the high plasma density region and 0.3 eV in the rest of the tunnel. It should be noted that an immediate consequence of such a low temperature in the tunnel is a very high resistivity of that background plasma, which, in turn, is equivalent to the near-vacuum condition for the magnetic fields that begin to rapidly expand into the tunnel immediately upon release.

The initial condition for the vector potential $A_0(r)$ prescribing the desired initial magnetic field configuration is generated by following the general prescription of [13] and numerically solving within the HiFi framework $\nabla^2 A_0 = -J_0$ for $A_0(r)$ given some current density source function $J_0(r)$ and subject to the boundary conditions $\hat{n} \times A_0 = 0$ and $\nabla \cdot A_0 = 0$, where $\hat{n}$ is the unit normal vector at the boundary surfaces. The following functional form is chosen for $J_0$:

$$J_0(r) = S(r) \lambda_{\text{sph}} \left[-\pi J_1(\alpha r) \cos(\pi z) \hat{r} + \lambda_{\text{sph}} J_1(\alpha r) \sin(\pi z) \hat{z}\right],$$

where $\lambda_{\text{sph}} = \sqrt{\alpha^2 + \pi^2}$, $\alpha = 3.8317$ is the first zero of the $J_1$ Bessel function, and
shown by Gray

no charge accumulation boundary condition on the electro-

5 kG and average density is 2

epoch ranges from 40 to 60

as in figure2(

flux-rope structure’s resistive and viscous dissipation. Clearly,

from 60

effects that degrade the magnetic fields and flows. The epoch

generally stable period of fully developed turbulence balanced

fluctuations most closely analyzed here. This time range sees a

formation/selective-decay epoch spans from 30 to 40

µ120

4. Results

The SSX produces flux-rope plasma that persists on the order of

120 µs. Figure 2 shows a time series of numerically integrated

B-field—\( |B| = \sqrt{B_x^2 + B_y^2 + B_z^2} \)—for the innermost tip

(1 cm from the cylindrical axis), interferometer line-averaged

number density, \( n \), Mach probe measurement of Mach number,

\( M \), and discharge current, \( I_{\text{dis}} \), for a sample shot and for an

average of 35 shots. For time analysis, the shots are divided

into epochs to account for the dynamical nature of the plasma

discharges as indicated by the dashed lines in figure 2. The

formation/selective-decay epoch spans from 30 to 40 µs. As

shown by Gray et al [3], the selective-decay of the plasma into a

Taylor state likely occurs in this time range. The equilibrium

epoch ranges from 40 to 60 µs and is the period of turbulent

fluctuations most closely analyzed here. This time range sees a

generally stable period of fully developed turbulence balanced

between the sourced plasma from the gun and resistive/viscous

effects that degrade the magnetic fields and flows. The epoch

from 60 µs on is considered the decay epoch and represents the

flux-rope structure’s resistive and viscous dissipation. Clearly,

as in figure 2(a), the magnetic field resistively decays away

by 120 µs.

In the equilibrium epoch, the average magnetic field is

5 kG and average density is \( 2 \times 10^{15} \text{ cm}^{-3} \); thus, radial and

axial Alfvén transit times are 0.3 µs and 3.5 µs respectively.

Ion temperature is measured using an ion Doppler spectrometer

system at the midplane. Background ion temperatures (i.e.
average temperatures made avoiding large heating events) are

on the order of 20 eV. Though not measured in this dataset,

previous measurements of the electron temperature suggest a

value of 10 eV for this system. Combined with the average field

and density values measured, estimates of ion gyrofrequency

\( (f_{\text{ci}}) \), ion gyroradius (\( \rho_i \)) and ion inertial length (\( \delta_i \)) can be

made. These values are listed in table 1. Of particular note,

the ratio of system size to \( \rho_i \) is large, \( R/\rho_i \approx 80 \), suggesting

that the influence of the wall on plasma dynamics is minimal.

The density trace in figure 2(b) shows peaking in the

formation epoch, but an eventual drop to approximately \( 1 \times

10^{15} \text{ cm}^{-3} \) for the majority of the discharge. The initial peaking

is likely due to the initial spheromak formation, where plasma

is being pushed into the tunnel, but before the threshold for

break-off is achieved. The Mach number trace is on average

positive, indicating flow predominately away from the gun

source as would be expected for a single source configuration.

The average Mach number during the equilibrium epoch is

\( M = 0.4 \) which for the sound speed \( C_s \) listed in table 1 yields

an average flow speed of 12 km s\(^{-1}\). It is likely that flow speeds

along the center axis can be even higher. Indeed, a time of flight

\[
S(r) = S_t(r) \times S_z(r),
\]

\[
S_t(r) = \begin{cases} 1, & r < (1 - \lambda J), \\
\frac{1}{2} \left[ 1 - \cos \left( \frac{1 - r}{\lambda J} \pi \right) \right], & (1 - \lambda J) \leq r \leq 1,
\end{cases}
\]

\[
S_z(z) = \begin{cases} 1, & \lambda J < z < (1 - \lambda J), \\
\frac{1}{2} \left[ 1 - \cos \left( \frac{1 - z}{\lambda J} \pi \right) \right], & (1 - \lambda J) \leq z \leq 1,
\end{cases}
\]

is the shape function that localizes the current density

source function within the high-density plasma region to

represent a single 1:1 aspect ratio Bessel-function spheromak

configuration modified so that \( J_0 \) drops to 0 at its edges over

the width \( \lambda J = 0.1 \). To break axisymmetry, an additional small

tilt-like perturbation is introduced into the vector potential at

t = 0 such that

\[
A(r)|_{r=0} = A_0(r) \left[ 1 + \delta r \cos(\theta) \exp(-z^2) \right],
\]

with \( \delta = 0.1 \).

All transport coefficient other than resistivity are set to

constant and uniform values throughout the simulations with

\( \kappa = 10^{-2}, \mu_i = 10^{-2}, \mu_e = 10^{-3}, \) and \( D_p = 10^{-4} \). Free-slip

hard wall boundary conditions are imposed on the ion flow

with \( \hat{n} \cdot \nabla (\hat{n} \cdot v_i) = 0 \) and \( \hat{n} \cdot v_i = 0 \), the diffusive thermal

and density transport across the boundary is limited by setting

\( \hat{n} \cdot \nabla T = \hat{n} \cdot \nabla \rho = 0 \), and the perfect conductor with

no charge accumulation boundary condition on the electro-
magnetic fields is set by requiring \( \hat{n} \times A = \hat{n} \times J = 0 \) and

\( \nabla \cdot A = \nabla \cdot J = 0 \).
Table 1. MHD wind-tunnel plasma parameters during the equilibrium epoch for the present configuration of SSX.

<table>
<thead>
<tr>
<th>$V_a$</th>
<th>$f_A$</th>
<th>$\tau_{Ax}$</th>
<th>$\tau_{A\rho}$</th>
<th>$\rho_i$</th>
<th>$\delta_i$</th>
<th>$C_s$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>240 km s$^{-1}$</td>
<td>7.6 MHz</td>
<td>3.5 $\mu$s</td>
<td>0.3 $\mu$s</td>
<td>0.091 cm</td>
<td>0.51 cm</td>
<td>31 km s$^{-1}$</td>
<td>0.0967</td>
</tr>
</tbody>
</table>

Figure 3. Wavelet power spectrum of $\dot{B}_\theta$ as a function of time for a single shot (same as that shown in figure 2).

estimate based on the axial separation distance between the interferometer measurement and the midplane $B$ measurement suggests a bulk plasma flow of 20 km s$^{-1}$. As seen in the single shot trace in figure 2, there can sometimes be a flow reversal which likely indicates a reflection of flow off of the far axial boundary.

4.1. Spectra

Given the dynamical nature of these plasmas, the spectral decomposition has been achieved using a Wavelet technique rather than with fast Fourier transforms (FFTs). This wavelet technique has been shown to be useful in situations where data may be non-stationary and provides a more accurate method for simultaneous spectral and temporal decomposition than a windowed Fourier transform because it applies a transform at many scales rather than a single scale [22]. To achieve better resolution in frequency space, a sixth-order Morlet mother wavelet has been used. Morlet wavelet scales (i.e. frequencies) are generally well matched to Fourier scales, though it has also been shown that the choice of wavelet is not critical for determining power spectral densities (PSD) [22].

A wavelet decomposition for a single shot $\dot{B}$ time series is shown in figure 3 with the wavelet scale converted to a Fourier frequency as the $y$-axis and time as the $x$-axis. The color scale corresponds to the normalized fluctuation power. Changes in the spectral nature can be seen with time: higher frequency fluctuations grow up and peak around 30 $\mu$s, hold stable until about 60 $\mu$s, and then begin to dissipate. This change in fluctuations supports the division of the data into epochs.

The shot-averaged spectra for the magnetic field components ($B_r$, $B_\theta$, $B_z$) are nearly identical suggesting that magnetic fluctuations are isotropic. For clarity, only the $B_\theta$ component is shown in figure 4. A similar approach is taken for density and Mach number, though without the frequency scaling.

As figure 4 shows, most of the power is concentrated at low frequencies, less than 100 kHz. It is thought, based on the energy arguments discussed in [3], that the large scale, low frequency magnetic fluctuations arise from the instability of the spheromak itself as it relaxes into the lower-energy Taylor state. This injected energy then cascades down into the smaller scales, higher frequency regions of the spectrum. The injection of flow energy may also be related to this process. However, the focus here is on the nature of the spectra beyond this energy.
injection scale and investigation into the nature of the energy injection is left for future work.

All three curves shown have roughly linear behavior in the spectral region between 50 kHz and 10 MHz. A power-law fit of spectral power to frequency \( (P(f) \sim f^{-\alpha}) \) is made for density and Mach number in this frequency range between 50 kHz and 10 MHz using a maximum likelihood estimation method (MLE) \[23\] yielding exponents of \( \alpha = 1.57 \) for the density spectrum and \( \alpha = 1.62 \) for the Mach spectrum, which serves as a proxy for the velocity spectrum in this experiment. An exponent of 1.62 for velocity is very close to the typical Kolmogorov exponent of \( 5/3 \approx 1.66 \) for hydrodynamic flow fluctuations.

The magnetic field spectrum appears to have two separate regions: a shallower sloped region between 50 kHz and 1 MHz and a steeper sloped region between 1 and 10 MHz. MLE fits for these regions give exponents of \( \alpha = 2.47 \) (shallower) and \( \alpha = 4.36 \) (steeper). Both the plots and the fits indicate that the spectra index for \( B \)-field fluctuations is steeper than that for Mach (velocity) fluctuations and thus steeper than that measured in the solar wind. This is perhaps indicative of a magnetic field dissipation mechanism that is present in the experimental plasma, but not in space.

The origin of the break in the spectra may come about in a number of ways. As suggested earlier, it is not likely an issue with probe frequency response as the drop in observed power exceeds the calibrated estimates of droop due to frequency response. First, it may simply be due to the effect of flowing plasma across the probe. The Taylor hypothesis—frozen-in flow—suggests that measured temporal spectra can be mapped to spatial spectra when the medium containing the fluctuations is moving past a probe at some velocity. Thus, in the spectrum, there is some critical frequency below which fluctuations can be considered to be predominately due to spatial structure, and above which fluctuations are due to some combination of spatial and temporal structure. A rough estimate for this critical frequency can be made by considering the average flow and probe size. The average axial flow for the equilibrium epoch as indicated by the Mach probe and the assumed \( C_s \) is 12 km s\(^{-1}\); with a 3 mm probe size, the critical frequency would be 4 MHz. Thus, the break in the slope could be showing a transition between the power spectrum representing the energy transfer in both spatial and temporal scales (higher frequencies) and a spectrum due to the energy transfer in only spatial scales (lower frequencies), since lower frequency temporal fluctuations become less observable by the probe.

The break could also be related to the temporal decorrelation of the signal. Figure 5 shows the autocorrelation function for \( B \) for each of the three axes, again during the equilibrium epoch. A decorrelation time can be defined as the \( \tau \) at which the autocorrelation function crosses zero. For \( \dot{B}_x \) and \( \dot{B}_y \), \( \tau = 1.0 \mu s \) which corresponds to a frequency of 1 MHz—also called a decorrelation rate. If there are any wave modes in the plasma, they could influence the spectrum at frequencies below the decorrelation rate. Indeed, there has been some evidence that mode types can have an effect on energy transfer rates that manifest in the power spectrum \[24\]. Above the decorrelation rate, the fluctuations arise purely from the turbulence which may have a different energy transfer rate. Consequently, this may appear as a change in the spectral index around the decorrelation frequency.

Finally, this break may be due, of course, to a transition to the dissipation range of the spectrum. Possible dissipation scales include the ion gyroradius and ion inertial lengths which are \( \sim 0.1 \text{ cm} \) and \( \sim 0.5 \text{ cm} \), respectively, for the equilibrium epoch. Dissipation could also occur directly in frequency space with coupling to the ion gyrofrequency, which here is 7.6 MHz. If the Taylor hypothesis is made, and frequencies are mapped to wavelengths using the average velocity, the break point occurs at a length of 1.2 cm; the ion inertial length scale maps to a frequency of 2.4 MHz.

Synthetic diagnostics of the HiFi simulations of the SSX MHD wind tunnel also produce time series \( B \)-field, density and flow measurements, with the diagnostics location and probe sizes designed to resemble those used in the experiment as much as possible. For example, \( B \)-field at the midplane is measured by taking a volume average over a 0.3 cm diameter sphere corresponding to the size of the experimental probes; while the axial location of the chord-averaged plasma number density diagnostic, as well as the radial location of the midplane flow measurement, correspond to the locations of the respective experimental measurements. However, unlike the experimental data, the simulated discharge consists of only a single spheromak that undergoes expansion, Taylor relaxation and decay with a lifetime on the order of 50 \( \mu \text{s} \). The simulation does not presently model slow formation, or the sustainment of plasma injection seen in the experiment; however, a comparison between the two can be made by observing the similarity of the simulated pulse with the tail end of the experimental discharge. This can be clearly seen in figure 6 where the simulated time series has been shifted to approximately match the decay features of the \( B \)-field, as well as a few fluctuation features of the density and Mach number.
The experimental shot shown in figure 6 was chosen for its particularly well-matched features.

This similarity highlights the assertion that despite the longer injection time of plasma in the experiment (and thus, less well-defined large scale structure), the selective decay and self-organization still occurs. Moreover, the similarity allows us to compare the turbulent features of the experiment and the HiFi simulations.

Figure 7 shows wavelet spectra for the simulation runs as well as the averaged experimental spectra. Due to the smaller number of points in the simulation time series, the wavelet decomposition used a fourth-order Paul mother wavelet, which has a slightly better time resolution than the sixth-order Morlet wavelet used above. The experimental data is also reanalyzed using the Paul wavelet for direct comparison to the simulation data. (Note that the differences between the two wavelet transforms is minimal for the experimental data, which is sampled at a higher frequency.) Like the experimental spectra, there appear to be two linear regions in the simulation spectra. The slopes of both the resistive-MHD and Hall-MHD simulation spectra are very close to that of the experimental data in the region between 20 and 300 kHz. The simulation spectra, however, begin to steepen at a lower frequency than the experimental data: $\sim 300$ kHz compared to $\sim 1$ MHz. This is consistent with the autocorrelation function for the simulation which gives decorrelation times on the order of 3.3 $\mu$s. The simulations do not appear to shed any light on the possible role of dissipation. The spectra for both resistive-MHD and Hall-MHD simulations are similar, so that any modification of the underlying dynamics due to the inclusion of the Hall physics is not apparent in the data. Nevertheless, the slope of the simulation spectra are very close to the experiment.

4.2. PDFs

While spectra can be useful for obtaining the relative rate of energy transfer amongst injection, inertial and dissipation scales, they can obscure other signatures of turbulence that can only be seen when looking at higher integral moments. One technique for investigating these higher moments is to construct PDFs from the data at different timescales and then to directly calculate the moments. The variation in the moments of PDFs as a function of timescale has been shown to be linear for fully developed fluid turbulence [25]. One can introduce a timescale into the time series by filtering [25, 26] or by taking differences of the data points at different time steps [10, 11]. This latter technique is used here, where a series of $\Delta B_\tau (t)$ increments is constructed for a variety of timesteps, $\tau$, as in

$$\Delta B_\tau (t) = \dot{B}(t + \tau) - \dot{B}(t) \quad (13)$$

A series of these PDFs for the equilibrium epoch time series is shown in figure 8 for $\tau$ of 0.15 $\mu$s, 0.75 $\mu$s, 3 $\mu$s and 15 $\mu$s which corresponds to filtering the time-series above frequencies of 6.6 MHz, 1.3 MHz, 333 kHz and 66 kHz respectively. The PDFs are normalized to the root mean square value of the increments and to the total integral of the histogram. A Gaussian function is fit to each distribution. One can clearly see a transition from non-Gaussian distributions for small $\tau$s to more Gaussian distribution for larger $\tau$s. The features of the non-Gaussian distributions are indicative of intermittency in the signal; the fat tails signify large swings in the values of the time series and the timestep indicates the relative temporal size of the intermittent events.
The level of intermittency as a function of $\tau$ can be quantified using a calculation of the flatness defined as [25]

$$F(\tau) = \frac{S_4(\tau)}{S_2(\tau)^2},$$

where $S_p(\tau)$ is the $p$th order moment for the distribution of increments, $\Delta B$, as in

$$S_p(\tau) = \int_{-\Delta B_{\text{max}}}^{\Delta B_{\text{max}}} P(\Delta B)(\Delta B)^p \, dx = \langle (\Delta B(\tau))^p \rangle$$

with $x$ representing a histogram bin width. An exact Gaussian distribution would have a flatness equal to 3. Note that as equation (15) indicates, the flatness can be equivalently calculated as a moment of the PDF or directly though an average of the time series of increments.

The resulting function for flatness is shown in figure 9 for each of the three $B$ measurements. The plot shows the trend toward Gaussian (as represented by the dashed line at $F = 3$) as the timestep is increased. An increase in flatness signifies an increase in the intermittency observed in the PDF—the flatness grows rapidly beyond $\tau = 1 \mu s$ corresponding to a frequency of 1 MHz. Since fluctuations beyond this frequency are more and more temporally correlated (as indicated by the autocorrelation function), it is clear that the intermittent nature is truly embedded in the magnetic signal. Conversely, a trend toward a Gaussian distribution might be expected for fluctuations that are not temporally correlated; however, recent work in this vein on solar wind measurements has speculated that such a change in flatness versus timescale might be expected at the transition from inertial to dissipation range turbulence [26]. It is also worth noting that the transition occurs around the autocorrelation time measured above. This is perhaps not unlike the spatial PDF measurement conducted in Greco et al [10] which showed a tendency toward Gaussian distributions beyond the radial correlation length whose analogue here would be the autocorrelation time.

The simulation data can also be analyzed within the PDF of increments framework. The flatness curve is also shown in figure 9 for the two different simulation runs. The shape of the curve is very similar to the experimental one, though the changes in flatness are offset in time increment as higher values of flatness are reached at larger increments. The largest values of flatness for both the experiment and the simulation are similar, suggesting that the mechanism responsible for the intermittency might be common to both.

5. Conclusion

We have hypothesized that the turbulent relaxation of a twisted flux-rope plasma in the SSX wind tunnel was promoted by intermittency and coherent structures. The rapidity of the relaxation process (about one Alfvén time) could be due to
fast local relaxation of the coherent structures followed by a slower relaxation of the structure as a whole. In this study, we have focused on time series and temporal frequency statistics: autocorrelation function, power frequency spectra, and PDFs of temporal increments. Spatial correlations and spectra will be reported in a later article.

Magnetic field power frequency spectra from the experiment show power-law behavior over two decades but with steeper spectral indices than measured in the solar wind and predicted from theory. Comparisons of power frequency spectra with numerical simulations show good correspondence with indices $\alpha \approx 2.5$ at low frequencies and steeper at higher frequencies. The reason for the break in the spectra slope has not been identified; however, both experimental and simulation data had measured decorrelation rates which could be placed near the respective break points. Furthermore, it is not clear whether a dissipation scale has been observed in the SSX plasma in these experiments.

Probability distribution functions of the magnetic field increments expose non-Gaussian higher order statistics connected to intermittency and coherent structures. We find large values of flatness at short time increments corresponding to small spatial scales in both the experiment and the simulations. In addition, we find that the shape of the curve of flatness with increasing time increment is also very similar in both the experiment and the simulations.

The observation of non-Gaussian values for flatness in the PDFs, and the trend for increasing flatness with decrease in timestep is a strong indication for the presence of intermittent fluctuations and/or coherent structures in the plasma, which could not be observed with spectra alone. The physical origin of these coherent structures in the SSX plasma has not yet been identified. As seen in previous simulation work [28], the coherent structures can be matched to the presence of current sheets which in turn could be the site of a reconnection layer. Given the observation of reconnection in past SSX work [29, 30], there is a strong likelihood that such events are present in the current SSX configuration though perhaps at a smaller scale.

Acknowledgments

We gratefully acknowledge many useful discussions with William Matthaeus. This work has been funded by the US DoE Experimental Plasma Research program and the National Science Foundation. The simulations were performed using the advanced computing resources (Cray XC30 Edison system) at the National Energy Research Scientific Computing Center.

References


and solar wind turbulence: scale-dependent kurtosis
Astrophys. J. 744 171

[27] de Wit T D, Alexandrova O, Furno I, Sorriso-Valvo L and
Zimbardo G 2013 Methods for characterizing microphysical
processes in plasmas Space Sci. Rev. 178 665–93

[28] Servidio S, Greco A, Matthaeus W H, Osman K T and
Dmitruk P 2011 Statistical association of discontinuities
and reconnection in magnetohydrodynamic turbulence
J. Geophys. Res. 116 A09102

[29] Cothran C D, Brown M R, Gray T, Schaffer M J and
Marklin G 2009 Phys. Rev. Lett. 103 215002

Three-dimensional reconnection and relaxation of merging
spheromak plasmas Phys. Plasmas 17 102106