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### Non-Optional Projects: Mathematical And Ethical

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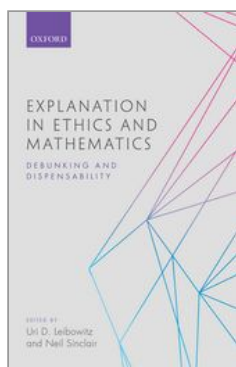
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## Explanation in Ethics and Mathematics: Debunking and Dispensability

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## Non-Optional Projects

### Mathematical and Ethical

Alan Baker

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### Abstract and Keywords

This chapter examines how the general framework for indispensability arguments developed by Enoch in the metaethical context plays out in its ancestral home, the philosophy of mathematics. Enoch's framework is inspired by the Quine-Putnam type of indispensability argument in mathematics and is liable to inherit the latter's holism. But once this holism is expunged from Enoch's framework it turns out that Enoch's indispensability argument is stronger in the moral than in the mathematical case, since it is more plausible that normative entities are indispensable to all projects of practical deliberation than it is that mathematical entities are indispensable to all projects of scientific theorizing. The upshot is that, given Enoch's framework, the move away from holism undermines indispensability arguments in mathematics but not in ethics

*Keywords:* deliberation, Enoch, indispensability, holism, mathematical objects, normative properties

### 12.1 Indispensability Theses

Indispensability arguments are widespread both inside and outside philosophy. Imagine that I make a New Year's resolution to learn how to cook French cuisine. Hearing of this project, you warn me that

(1) Shallots are indispensable for fine French cooking

and that this is a problem because the local grocery stores do not stock shallots. Presuming that this indispensability claim is true, what are my options for pressing ahead with my cooking project in the absence of shallots? The first

option is for me to simply lower my standards: *fine* French cooking may require shallots, but I will just set my sights on *adequate* French cooking, which should still be possible, perhaps by just substituting onions where shallots are called for. A second option is to stick with the fine cooking project, but to switch cuisines. Perhaps I will learn instead how to cook fine *Thai* food, which is shallot-free, and make use of the well-stocked Asian grocery store in my neighborhood. A third option is to keep my focus on fine French cuisine, but abandon my aspirations to cook it. Instead I will read about its history, browse classic cookbooks, and go out to upmarket French restaurants to sample canonical dishes.

All three of these options would permit me to retain important aspects of my original project, while not purchasing a single shallot. And I can do this without challenging the ‘shallot indispensability thesis’ enshrined in (1). But what if I want to stick to the letter of my original intention, namely to cook fine French food? Do the following three propositions form a consistent set?

- (p.221)** (1) Shallots are indispensable for fine French cooking.
- (2) I carry out the project of learning to cook fine French food.
- (3) I do not use any shallots.

I think that this set is indeed consistent, and that—despite the frivolity of this particular example—that there is a more general point here that has important implications for indispensability arguments in different areas of philosophy.

The two areas that I shall focus on here are philosophy of mathematics and metaethics. In recent years, several philosophers have sought to draw parallels between the use of indispensability arguments in these two areas. Although I think that there are useful insights to be gained through such a comparative study, my main goal in this chapter is to sound a note of caution. Differences between the contexts of applied mathematics and of metaethics mean that when we formulate sets of propositions corresponding to (1), (2), and (3) for each of these domains, the set is consistent in the mathematical case but not in the ethical case. This gives the opponents of indispensability an avenue of escape in the former that is not available in the latter, which in turn affects how the corresponding indispensability argument needs to be formulated. To put the point in terms that are not yet very helpful: applied mathematics is more like French cooking than metaethics!

An influential argument for mathematical Platonism is based on the following indispensability thesis:

- (4) Mathematics is indispensable for science.

Put very briefly, the Quine–Putnam indispensability argument (QPIA) ties our ontological commitments to those of our best available scientific theories. We ought rationally to believe in theoretical posits such as electrons and black holes because they are indispensable to current science. Abstract mathematical objects such as numbers, functions, and sets are also indispensable to current science. Hence we ought to believe in the existence of abstract mathematical objects, and we ought to embrace mathematical Platonism.

(1) and (4) are similar in form: in both cases, indispensability is a two-place relation, ‘X is indispensable for Y’. But there are apparent differences in what kind of relatum fills these X and Y slots in the respective relations. In our cooking example, what is claimed to be indispensable is a kind of *object* (i.e. shallots). In the Platonism example, what is claimed to be indispensable is a *subject matter* (i.e. mathematics). Actually, I think (1) and (4) can be paraphrased, without loss of meaning, so as to bring their respective X-relata more into line. The difference is that, in discussions of French cooking, no side in the debate doubts the **(p.222)** *existence* of shallots. Hence, involving shallots directly as a relatum in the indispensability claim in (1) does not beg any questions. By contrast, the relevant debate within the philosophy of mathematics is precisely about whether abstract mathematical objects exist. So anti-Platonists will not (initially) accept any claim about the indispensability of mathematical objects *per se*. (4) is standardly read as elliptical for

(4’) Quantification over mathematical objects is indispensable for science.

Thus, what is asserted to be indispensable for science is the *activity* of quantifying over mathematical objects, rather than the mathematical objects themselves. Correspondingly, (1) could also be paraphrased to put an activity as the X-relatum:

(1’) Using shallots is indispensable for fine French cooking.

So much for the first half of the indispensability relation.<sup>1</sup> What about the second half? In his recent book *Taking Morality Seriously*, David Enoch draws inspiration from QPIA to formulate an indispensability argument for what he calls ‘robust metanormative realism’. Central to Enoch’s discussion is the distinction he draws between ‘instrumental indispensability’ and ‘intrinsic indispensability’. Instrumental indispensability corresponds to the two-place relation we have been discussing above. As Enoch puts it, ‘indispensability is always indispensability *for* or *to* a certain purpose or project’ (2011 p. 67, italics in original).

What makes Enoch’s approach so interesting is the *generality* of the framework that he constructs for analyzing indispensability arguments across a range of different contexts. My plan is to look at some key features of Enoch’s framework

and see what insights might be applied back to more specific indispensability arguments, both in mathematics and in metaethics.

### 12.2 Projects

Despite the importance to Enoch's analysis of the notion of a *project*, nowhere does he give an explicit definition of what he means by this term. He does, however, give several examples of candidate projects. His two core examples are the *scientific project* and the *deliberative project*. Enoch does not say much about the scientific project, other than that he sees it as filling the Y-relatum in the indispensability thesis of QPIA. He goes into considerably more detail about the **(p.223)** deliberative project, since this lies at the core of his own version of the indispensability argument. Deliberation, for Enoch, is 'the process of trying to make the decision it makes the most sense for one to make' (2011 p. 73). In deliberating, 'you *commit* to there being (normative) reasons relevant to your deliberation' (p. 74). The deliberative project, then, is the general practice of 'asking ourselves what to do, what to believe, how to reason, what to care about' (p. 70). In addition to these two examples, Enoch also mentions in passing the following projects: 'the reasoning project' (p. 64); 'the project of finding out about [the external world]' (p. 64); 'the project of sorcery' (p. 69); 'the project of achieving eternal bliss' (p. 69).

These examples of projects are strikingly diverse, but one feature they have in common is their extreme *generality*. As we shall see, this generality plays an important role in the next stage of Enoch's argument. As Enoch points out, instrumental indispensability *per se* cannot be enough to ground ontological commitment: some restriction is needed on what counts as an 'acceptable' project. What places the scientific project on the right side of this divide, and the project of sorcery on the wrong side? Enoch has an interesting answer to this question. He introduces a second notion, *intrinsic indispensability*, and defines a project to be intrinsically indispensable if it is 'rationally non-optional'. This results in the following proposed criterion of ontological commitment:

(IP) We ought rationally to be ontologically committed to F's if F's are instrumentally indispensable for an intrinsically indispensable project.

Thus, we ought to believe in the existence of electrons, and numbers, because they are instrumentally indispensable to the intrinsically indispensable scientific project. And, according to Enoch, we ought to believe in irreducibly normative truths because they are instrumentally indispensable to the intrinsically indispensable deliberative project. As Enoch puts it, 'the respectability of the project confers respectability on [the] commitment' (2011 p. 69). Among the projects that Enoch mentions, all very general in their scope and scale, some are rationally non-optional and others are not. The project of sorcery, for example, is presumably one that we can (and probably should) opt out of. Hence it does not

matter to our ontological commitments what is (instrumentally) indispensable for sorcery.

(IP) has an appealing symmetry, in its combining two notions of indispensability into a single criterion of ontological commitment. And the idea of using rational non-optionality as the marker of ontologically serious projects is an interesting one. So let us assume, for sake of argument, that (IP) is correct as a criterion for ontological commitment. In making this assumption, I am **(p.224)** deliberately bypassing at least two significant debates within the indispensability literature. Firstly, many anti-realists take issue with instrumental indispensability being sufficient, even in the context of an intrinsically indispensable project. So-called ‘weasel nominalists’ in the philosophy of mathematics take this line against QPIA, arguing that there is nothing irrational about making certain claims in the course of pursuing a project and then taking back these claims at the end. Secondly, some anti-realists have questioned the legitimacy of linking intrinsic indispensability to truth. Just because I cannot rationally avoid engaging in a project, why think that the results of the project are likely to be true? There is plenty more to be said on both sides of these two debates, but they will not be my concern here.

The question I want to address in the remainder of the chapter is whether Enoch’s framework can be applied to more recent indispensability-centered debates in the philosophy of mathematics, and in particular whether his notion of a ‘non-optional project’ can get traction in these debates. One irony of Enoch’s drawing on indispensability debates in the philosophy of mathematics to motivate his favored version of metanormative realism is that in many respects these debates have moved on in ways that have a significant impact on Enoch’s analytical framework. One major change, as we shall see, is that the focus has shifted—in these philosophy of mathematics debates—to projects that look very different from the large-scale projects that Enoch presents.

### 12.3 Abandoning Holism about Projects

What does it mean for a particular activity to be indispensable ‘for science’? As Enoch points out, little attention was paid to this question in the early debates over QPIA, mainly because this argument is firmly rooted in Quine’s holism. On this picture, science is a web of interconnected theories evaluated as a single whole by balancing such criteria as empirical adequacy and simplicity. If the best such web contains mathematical posits, then mathematics is indispensable for science. It matters not where in the web such posits appear, nor what precise theoretical role they play.

Over the past decade or so, however, defenders of Platonism have sought to separate the indispensability argument from Quinean holism. Once holism is abandoned, specification of the Y-relatum of the indispensability thesis becomes crucial—vague reference to indispensability ‘for science’ is no longer enough,

since it may not necessarily be the case that all parts and aspects of the scientific project are ontologically on a par. There are at least two basic axes along which such specification might take place. Firstly, we might subdivide the scientific **(p.225)** project into different kinds of *activity* (or theoretical role), such as description, prediction, explanation, or theory formulation. Secondly, we might subdivide by *subject area*, for example chemistry, physics, evolutionary biology, or quantum mechanics.<sup>2</sup>

Holism-free versions of QPIA have tended to focus on specification of theoretical role. The most popular of these ‘second-generation’ indispensability arguments focuses on *explanatory role*. The motivating idea is that the target audience for QPIA is scientific realists, and that the core generator of ontological commitment within scientific realism is *inference to the best explanation*. We believe in the existence of electrons not merely because we cannot avoid quantifying over them, but because the existence of electrons best explains various observations that we make. What matters, therefore, is whether mathematical objects play an indispensable *explanatory* role in science. Thus the sharpened indispensability thesis is:

(5) Mathematics plays an indispensable explanatory role in science.

The resulting argument is known variously as the Explanatory Indispensability Argument or the Enhanced Indispensability Argument (EIA).

This move towards focusing more explicitly on explanation, as embodied in EIA, may not seem much different in spirit from Enoch’s own explication of QPIA. According to Enoch’s analysis, although QPIA is focused on the scientific project as a whole, the principal evaluative criterion used when comparing competing theories is explanatory. F’s are indispensable for the scientific project if the most explanatory overall theory includes F’s. However, this common focus on explanation masks an important difference between EIA and Enoch’s conception of QPIA. This is because Enoch retains the fundamental holism that is present in the Quinean framework; competing formulations of a single overarching scientific theory are still compared as wholes. All that has changed is that the theoretical virtue of explanatoriness has been promoted to the top spot when comparing these theories.

EIA is sharply different in this respect, since—as has already been mentioned—it is predicated on a rejection of Quinean holism. No longer are scientific theories (or webs of scientific theories) compared in their entirety. Instead, the focus is on *individual* explanations in science. The point is that what matters is the explanatory role of the posits, not the overall explanatoriness **(p.226)** of the theory in which they happen to be embedded. The proponent of EIA concedes the point—pressed by Maddy, Melia, and others—that mere appearance in an explanatory theory is not enough to ground ontological commitment to a given

posit. Our best overall theory of fluid mechanics may make reference to such things as continuous fluids and infinitely deep reservoirs, but that ought not to commit us to the existence of idealized entities of this sort.

Couched in Enoch's terminology, what has happened in the evolution from QPIA to EIA is a shift from viewing the instrumental indispensability of mathematics as pertaining to the scientific project as a whole, or even just to the explanatory subproject within science, and instead viewing this instrumental indispensability as pertaining to a collection of 'mini-projects' that concern the explanation of specific scientific phenomena. This shift is crucial because of the following fact about scientific explanation: *many (perhaps most) scientific explanations do not make explanatory use of mathematics*. For many of these mini-projects, therefore, mathematics is not instrumentally indispensable. For those explanatory mini-projects that do make indispensable use of mathematics, we need to revisit the question of whether they are indeed rationally non-optional in Enoch's sense.

### 12.4 Explanatory Mini-Projects—Optional and Non-Optional

For present purposes, I shall take an *explanatory mini-project* to be an investigation that seeks to answer a request to explain some specific scientific phenomenon or pattern of phenomena. Such investigations may have both theoretical and experimental components, and may vary considerably in their scope and sophistication. To give us something to focus on, let us consider two such potential mini-projects:

(6) Why do periodical cicadas have prime periods?

(7) Why does *C. Elegans* have a prime number of cells?

Mini-project (6) is quite well-known, and has been much discussed in the recent literature on EIA.<sup>3</sup> Mini-project (7) has never (to my knowledge) been either articulated or pursued.

So how does the issue of rational non-optionalness play out in the context of EIA? It is tempting to think that the rational non-optionalness of the scientific **(p.227)** project is enough to ground EIA also, since all the explanatory mini-projects are components of the scientific project. But this is not enough. For we might be able to engage in the (non-optional) scientific project to a sufficient degree without engaging in those explanatory mini-projects that indispensably involve mathematics. Returning to my initial French cooking scenario, we can now see more clearly how the three core claims, listed below, could be jointly consistent:

(1) Shallots are indispensable for fine French cooking.

(2) I carry out the project of learning to cook fine French food.

(3) I do not use any shallots.



Shallots are indispensable for fine French cooking, but many individual French dishes do not call for shallots. So it is in fact possible to pursue the project of learning to cook fine French food, at least to a considerable degree, without acquiring any shallots. I can do this by picking those mini-projects, within the overall project of French cuisine, that do not involve shallots.

In this respect, there is a crucial disanalogy between the scientific project and the deliberative project. Amongst the mini-projects that make up the scientific project, some involve mathematics and some do not. By contrast, all—or nearly all—of the mini-projects that make up the deliberative project involve reference to metanormative truths. This is because, for Enoch, a deliberative mini-project is precisely the search for, and evaluation of, normative reasons that are relevant to the given decision. (If no reasons are forthcoming one way or the other, Enoch refers to the process not as ‘deliberation’ but as ‘arbitrary picking’ (2011 p. 73).) In other words, reasons are built into every specific case of deliberation, whereas mathematics is not built into every specific case of scientific explanation. As a result, it is plausible—in the metanormative context—to give a strong reading to the claim of instrumental indispensability: all (or nearly all) mini-projects within the deliberative project make essential reference to metanormative truths. Hence we cannot pursue the deliberative project *at all* without engaging with such truths. And this is the case regardless of whether any given mini-project is intrinsically indispensable.

As I have argued, the situation with respect to scientific explanation (and French cooking!) is sharply different. Since within science there are many explanatory mini-projects that avoid commitment to mathematical objects, the issue of the intrinsic indispensability of these mini-projects now comes to the fore. Put simply, unless at least some of those explanatory mini-projects that involve mathematics are themselves intrinsically indispensable, the road to ontological commitment to mathematical objects is blocked. (Analogously, what is required to force a commitment to acquire shallots is not just the **(p.228)** demonstration that some French recipes need shallots, but that those dishes are themselves an unavoidably central part of French cuisine.)

Why is this a problem for the defender of EIA who wants to use Enoch’s framework? The problem is that specific mini-projects *do* seem to be rationally optional, and hence not intrinsically indispensable.<sup>4</sup> Firstly, it seems psychologically possible to resist pursuing a specific explanatory project, even if we cannot stop ourselves across the board from seeking explanations of worldly phenomena. If someone poses the question of why cicadas have prime periods, there seems nothing to stop us from simply refusing to take up this challenge.<sup>5</sup> Secondly, this point seems to generalize across mini-projects regardless of context. As Enoch himself notes, ‘We can, of course, stop deliberating about one thing or another...It’s opting out of the deliberative project as a whole that may not be an option for us.’ (2011 pp. 70–1).<sup>6</sup> If, as seems to be the case, rational

non-optionalness of this psychological sort does not work to underpin a notion of intrinsic indispensability once we move from projects to mini-projects, is there some other sense of non-optionalness that can do this job? One idea is to draw on methodological considerations from the scientific project itself. In other words, might there be rational grounds, *given* the general pursuit of the scientific project, that compel engagement with certain mini-projects?

Let us schematize the notion of explanatory mini-project that was introduced at the beginning of Section 12.4, and view it as stemming from a request to explain why some type of physical phenomenon, P, has property, Q. Rather than asking when we ought rationally to pursue a mini-project of this sort, it makes more sense to ask instead about criteria for when it is rationally acceptable *not* to pursue it. Below is a list of some putative such criteria:

- (a) Truth: There are scientific grounds for doubting the truth of the explanandum, i.e. that P does in fact have property Q.<sup>7</sup>
- (b) Relevance: P and/or Q are not part of the subject matter of science.<sup>8</sup>
- (p.229)** (c) Significance: The fact that P has property Q is not scientifically significant.<sup>9</sup>
- (d) Feasibility: The mini-project to explain why P has property Q has little chance of success.<sup>10</sup>

Note that (c) and (d) have more to do with practical rationality than theoretical rationality. In other words, if the explanandum is both scientific in its subject matter and such that we are justified, on scientific grounds, in believing it, then the only grounds for not engaging in the mini-project are pragmatic.

### 12.5 Picking Out Physical Patterns

Things get more complicated, however, if we shift our attention to one particular form of explanandum that is exhibited by many putative mathematical explanations in science. Both of the explanatory mini-projects listed in Section 12.4 have the following form:

(MP) Why does physical phenomenon, P, have mathematical property, M?

For example, mini-project (7) asks why *C. Elegans* has a prime number of cells. A natural way of paraphrasing this claim is that the cells of *C. Elegans* can be mapped one-one onto a set whose cardinality is a prime number. However, here the explanandum is itself specified using objects from the contested (i.e. mathematical) domain. This appears to open the way to a third way of avoiding engagement with a given mini-project, on theoretical rational grounds, namely that the explanandum is only true if the existence of objects from the contested domain is already assumed. The point is that it looks question-begging to try to give an argument for the existence of F's based on the indispensability of F's for a mini-project which is itself premised on the existence of F's!

The problem, for the defender of mathematical Platonism, is that many—perhaps most—putative examples of mathematical explanation in science pertain to explananda that have the form given by (MP) above. Is there a way to argue, non-question-beggingly, for the rational non-optionality of such mini-projects? My view is that the Platonist does have an answer to the above challenge. I shall begin by sketching the general contours of the Platonist strategy and then show how it plays out in some specific cases.

**(p.230)** The first step is the use of a mathematical property, *M*, to pick out a feature of some token physical phenomenon, *P*, or to pick out a pattern that holds across a range of physical phenomena of a given type. The key point is that this purely *descriptive* use of *M* is not taken to be ontologically committing by either side in the EIA debate, even if it is indispensable. Therefore, it does not beg the question against the anti-Platonist side to take this claim, that *P* has property *M*, as a provisional explanandum. We only get ontological commitment if the pattern picked out by *M* has *explanatory power*, and this depends on what the best explanation of (MP) turns out to be. I will not try here to give a precise characterization of explanatory power, but I take it to be closely linked to the capacity to *generalize* across a range of different situations. If the property or pattern picked out by *M* can in turn explain various more specific properties and patterns, then this boosts its explanatory credentials. Conversely, if nothing of much significance follows from possessing *M* then the mathematical property is not explanatory.

I shall formalize the above line of reasoning by introducing the notion of a mathematical property being ‘salient’:

Definition: *M* is *salient* with respect to *P* if there is no other mathematical property, *M\**, such that *M\** is part of the best explanation of why *M* applies to *P*, but *M* is not part of the best explanation of why *M\** applies to *P*.

The meta-property of salience is intended to pick out those mathematical properties that are explanatorily significant. It also provides a criterion of intrinsic indispensability: an explanatory mini-project of the form (MP) is intrinsically indispensable if and only if *M* is salient with respect to *P*.

To see how this plays out in an actual example, let us return to mini-project (6), which aims to explain why periodical cicadas have prime periods. As discussed in a previous paper, the best explanation of this fact is the following (cf. Baker 2005 p. 233):

(6a) Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous. [Biological ‘law’]

(6b) Prime periods minimize intersection (compared to non-prime periods). [Number theoretic theorem]

(6c) Hence organisms with periodic life-cycles are likely to evolve periods that are prime.

My claim is that primeness here is a salient mathematical property. On the one hand, there seems to be no other mathematical property whose holding explains **(p.231)** why primeness holds. On the other hand, the above explanation can be extended to explain why other more specific mathematical properties hold. For example, we can use it—together with some specific ecological facts—to explain why a given cicada subspecies has a seventeen-year period:

(6d) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from fourteen to eighteen years.

(6e) Hence cicadas in ecosystem-type, E, are likely to evolve seventeen-year periods.

A significant feature of this specific explanandum, (6e), is that its mathematical component is dispensable. Reference to the period having a length of seventeen years can be paraphrased, in familiar fashion, into the language of first-order logic with identity. Thus the explanatory credentials of the salient mathematical property of primeness include the capacity to explain *non*-mathematical explananda.

It is important to note that salience is not an intrinsic feature of a given mathematical property. Just because primeness is salient in the context of mini-project (6) does not mean that it is salient in other contexts to which it applies. Consider, for example, mini-project (7), which is to explain why the nematode worm, *C. Elegans*, has a prime number of cells.<sup>11</sup> Experimental evidence tells us that *C. Elegans* has 1031 cells, and nothing in current biological theory gives us reason to think that there is any particular significance to the fact that this number is prime. Perhaps there is some explanation for why *C. Elegans* has 1031 cells, based on its evolutionary and developmental history. In this case, the best explanation for the explanandum of mini-project (7) has the following form:

(7a) [...Evolutionary/developmental explanation...]

(7b) Hence, *C. Elegans* has 1031 cells.

(7c) 1031 is a prime number.

(7d) Hence, *C. Elegans* has a prime number of cells.

My claim, therefore, is that—in the context of mini-project (7)—primeness is *not* a salient property. It is explanatorily superfluous.

Another way of looking at the contrast between (6) and (7) is in terms of the direction of explanation. The explanation given in (6) shows us that the cicada subspecies has period seventeen (in part) *because* its period is prime. Conversely, the explanation in (7) shows us that *C. Elegans* has a prime number of cells *because* it has 1031 cells. In (6), there is a ‘top-down’ explanation using **(p.232)** primeness; in (7) there is a ‘bottom-up’ explanation of primeness. Hence primeness is salient in the former context but not in the latter.

Another aspect of explanatory power that is not an explicit byproduct of salience is the capacity for a given pattern of explanation to apply to other, distinct contexts. The pattern of explanation in the cicada mini-project is quite general, but does this mathematical framework actual apply to other cases? One obvious place to look is for other kinds of periodical organism, however it is unclear from the biological literature whether there are any cases of organisms with fixed, multi-year lifecycles other than periodical cicadas. In the absence of a clear case from evolutionary biology, I offer here a mini-project from a quite different subdomain of the scientific project. Despite its very different setting, it shares the mathematical framework of the cicada mini-project and with it the salient property of primeness.

(8) Why do the rear cogs of brakeless, fixed-gear bicycles typically have a prime number of teeth?

Before presenting a candidate explanation for this fact, a couple of clarifying remarks are in order. A fixed-gear bicycle has neither a gear-changing mechanism nor a ‘freewheel’ on the rear axle, so the pedals keep rotating whenever the back wheel is rotating. While not especially practical, such bicycles are quite popular, especially with riders who want to perform tricks of various kinds. Because stopping the pedals stops the back wheel, there is technically no need for a separate hand-operated brake mechanism. When the rider stops pedaling, the back wheel stops turning and the bicycle skids to a halt. This produces wear on the back tire, and if the same part of the tire is repeatedly used in such skids then this produces what is known as a *skid patch*.

Here then is the explanation of (8):

(8a) Tires last longer if they have fewer skid patches.

(8b) The number of skid patches is maximized if the number of teeth on front and rear cogs are coprime.

(8c) Prime numbers are coprime with the fewest other numbers.

(8d) Cyclists prefer to maximize the life of their tires.

(8e) Hence, rear cogs are typically chosen to have a prime number of teeth.

The parallel with the cicada explanation is clear. In particular, the key mathematical component relates primeness to minimizing the frequency of intersection between two periodical phenomena (in this case the rotation of the front and rear cogs). The property of primeness is again salient, and it can be used to explain more specific facts about fixed-gear bicycles. For example, why does the rear cog (**p.233**) of this particular brakeless, fixed-gear bicycle have seventeen teeth? A candidate explanation might run as follows:

(8f) Front cogs with between forty and fifty teeth and a gear ratio between 2.3 and three are preferred by most cyclists.

(8g) Hence, the rear cog should have between fourteen and eighteen teeth.

(8h) Hence, this rear cog has seventeen teeth.

Here (8f) is an empirical fact about cyclists' preferences, and (8g) follows from (8f) using basic arithmetic.

To summarize, when a mathematical property is *salient* in a given mini-project then it will tend to exhibit explanatory power in both 'vertical' and 'horizontal' directions. In the vertical direction, it will explain other, more specific properties that hold in the context of the given mini-project. In the horizontal direction, it will tend to feature in parallel patterns of argument that apply to quite different mini-projects.

### 12.6 Avoiding Ontological Inflation

I have argued that it is important for the defender of the Explanatory Indispensability Argument in the philosophy of mathematics to have some account of what makes an explanatory mini-project intrinsically indispensable. Otherwise the anti-Platonist opponent can accept the instrumental indispensability of mathematics to the overall project of science, and just avoid those mini-projects in which mathematics is playing a specifically explanatory role. I have also argued that salience is the key marker for when an explanatory mini-project that involves a mathematical property is intrinsically indispensable. But there is also a second reason why it is important to distinguish the indispensable mini-projects from the dispensable ones, and this is to avoid over-generating mathematical ontological commitments.

We have actually already witnessed an example of such mathematical inflation, in mini-project (7). The best explanation of why *C. Elegans* has a prime number of cells makes reference to the (ineliminable) property of primeness. But the salient property here is having 1031 cells, and this can be eliminated using the language of first-order logic with identity. So including the non-salient property here would increase our mathematical commitments. In this instance it does not seem especially serious, of course, because primeness comes in as a salient property in other mini-projects, such as the cicada mini-project, (6), and the

skid-patch mini-project, (8). So our global mathematical commitments are not inflated. However this may not always be the case. For a more dramatic case of **(p.234)** potential mathematical inflation, consider the following explanatory mini-project:

(9) Why do North American cicada species have periods that correspond to the numerators of coefficients in the series expansion of  $\sinh x/\sin x$ ?

Here,  $\sinh x$  is the hyperbolic sine function, which can be thought of as the imaginary part of the ordinary trigonometric sine function when this is extended into the complex plane. A series expansion is a method for calculating a function that cannot be expressed just by elementary operators (addition, subtraction, multiplication, and division). The resulting series has an infinite number of terms, and the more terms that are included in a given calculation the more accurate the approximation to the original function. Further mathematical details are not relevant here, except to note that the numerators for the coefficients of the first three terms of the series for  $\sinh x/\sin x$  are 1, 13, and 17.<sup>12</sup>

The scenario here is similar to the *C. Elegans* mini-project, (7). As in that case, the link between the mathematical property and the physical phenomenon is (presumably) completely coincidental. If so, then the best explanation of (9) has the following form:

(9a) Cicadas species are either annual or they are periodical.

(9b) North American periodical cicadas have periods 13 and 17.  
[Explanation imported from mini-project (6).]

(9f) Hence, North American cicadas have periods 1, 13, and 17.

(9g) The initial numerators for the series expansion for  $\sinh x/\sin x$  are 1, 13, and 17.

(9h) Hence, cicada species have periods that correspond to the numerators of coefficients in the series expansion of  $\sinh x/\sin x$ .

The mathematical property of being a numerator of the given series expansion is not salient in (9). Why not? Because there is another mathematical property, having period 1, 13, or 17, which explains the holding of the series numerator property but is not explained by this latter property.

What is different about mini-project (9) is that the mathematical stakes are higher. Taking (9) to be a genuine target of explanation forces the introduction of hyperbolic functions. This expands the domain of mathematical ontology at

least from the natural numbers to the real numbers, since the function  $\sinh x$  is given by **(p.235)**

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

and perhaps to the imaginary numbers also, since complex analysis provides the most natural setting for hyperbolic functions such as  $\sinh$ .

### 12.7 Conclusions

In this chapter, I have focused on Enoch's proposed indispensability-based criterion for ontological commitment,

(IP) We ought rationally to be ontologically committed to  $F$ 's if  $F$ 's are instrumentally indispensable for an intrinsically indispensable project.

I have focused in particular on his notion of 'project' and what it might be for such projects to be 'intrinsically indispensable'. My main contention has been that the issue plays out very differently in the context of the two major domains that Enoch considers, namely applied mathematics and metaethics. Central to this difference is that metanormative truths permeate the deliberative project in a way that mathematical truths do not permeate the explanatory subproject of science. Once this is coupled with the general abandonment of strong holism assumptions in the philosophy of science, this shifts the debate to focus on individual 'mini-projects' rather than the global project that Enoch considers. For such mini-projects, a different notion of rational non-optionality is needed to ground their intrinsic indispensability. I argue that the notion of a *salient mathematical property* can do this job in the context of mini-projects that seek to explain why some physical phenomenon has a given mathematical property. Whether some corresponding notion of a *salient metanormative property* can do useful work in grounding a metaethical indispensability argument is a question that I leave to be addressed on another occasion.

### References

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### Notes:

(<sup>1</sup>) In what follows, I will mostly revert to the original formulations, (1) and (4), and not worry about paraphrasing the X-relatum along the lines of (1') and (4').

(<sup>2</sup>) In this connection, it is interesting to note that in specifying the project associated with mathematical indispensability arguments, Enoch sometimes describes it in activity terms (as 'the explanatory project') and sometimes in subject-matter terms (as 'the scientific project').

(<sup>3</sup>) See e.g. Baker (2005, 2009).

(<sup>4</sup>) A definitive defense of this claim is hampered by the fact that Enoch does not say much more about what it is to be 'rationally non-optional' (as Enoch himself acknowledges in a humorous footnote (2011 p. 71, n. 51)).

(<sup>5</sup>) Compare Hume-style assertions about the psychological inevitability of inductive reasoning. Even if this is true in general, we do seem capable of resisting specific cases of inductive inference.

(<sup>6</sup>) Compare also Enoch's remark: '[I]t's not clear that this line of thought can be applied to deliberation as a whole (rather than to some particular tokens of deliberation).' (2011 p. 77)

(<sup>7</sup>) For example, if the explanandum is that completing an Euler tour of some very complicated network is impossible. Perhaps it is possible after all!

(<sup>8</sup>) For example, if the explanandum is that the Mona Lisa is considered to be such a valuable work of art.

(<sup>9</sup>) For discussion of this issue in connection with mathematical explanation in science, see Baker (2012).

(<sup>10</sup>) Enoch discusses this issue (2011 pp. 61–3), and argues that having a low chance of success is good grounds for taking a given project to be optional.

(<sup>11</sup>) Another, even simpler, example: why do human beings have a prime number of legs?

(<sup>12</sup>) This series appears as sequence A069853 in the Sloane Online Encyclopedia of Integer Sequences.

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