2007

Monetary Perspective On Underground Economic Activity In The United States

R. D. Porter

Amanda Bayer
Swarthmore College, abayer1@swarthmore.edu

Recommended Citation

R. D. Porter and Amanda Bayer. (2007). 2nd. "Monetary Perspective On Underground Economic Activity In The United States". *The Underground Economies: Tax Evasion And Information Distortion.* 129-158.
https://works.swarthmore.edu/fac-economics/368

CHAPTER 5

Monetary perspective on underground economic activity in the United States

RICHARD D. PORTER
and
AMANDA S. BAYER

There are widespread reports of a growing underground, or unobserved, economy in the United States and in other countries. The unobserved economy seems to develop principally from efforts to evade taxes and government regulation. Although no single definition of such activity has been universally accepted, the term generally refers to activity – whether legal or illegal – generating income that either is underreported or not reported at all (see Chapter 1 in this volume). Some authors narrow the definition to cover income produced in legal activity that is not set down in the recorded national income statistics.[1]

Recent discussion of underground economic activity was stimulated by publication of two estimates, one by Gutmann (1977) and the other by Feige (1979), of the size of the underground economy in the United States; these estimates were derived from aggregate monetary statistics. In the ensuing years, numerous other estimates have been made of the underground economy in the United States and in other countries. The magnitude of some of these estimates has prompted congressional hearings and various government studies. In 1979, the Internal Revenue Service (IRS, 1979) estimated that, for 1976, individuals failed to report between $75 billion and $100 billion in income from legal sources and another $25 billion to $35 billion from three types of illegal activity – drugs, gambling, and prostitution. In a more recent study, the IRS estimated that unreported income from legal sources rose from $93.9 billion in 1973 to $249.7 billion in 1981 whereas unreported income from these same three illegal activities rose from $9.3 billion to $34 billion (IRS, 1983). To estimate unreported legal source income, the IRS mainly used individual taxpayer data from its Taxpayer Compliance Measurement Program – which audits a sample of income tax returns – and data from its Information Returns Program – which utilizes information from the

[1] By convention, the national income accounts do not include illegal activities such as loan sharking or trafficking in illicit drugs.

payers of income. Estimates of unreported income from legal sources for individuals not filing returns were developed from cross-checking information from two nationwide household surveys against the records of the Social Security Administration and the IRS. Estimates of unreported income associated with illegal activity were based on survey data and arrest records.

Proponents of the monetary statistics approach question the accuracy of estimates derived from such sources as administrative records and surveys, with the assertion that the methods employed are likely to lead to an understatement of actual unreported income. They believe that monetary statistics provide a better source for gaging underground activity. Gutmann (1977), for example, postulates that currency is the sole medium of exchange in the underground economy, and thus an increase in activity in that sector would be evidenced by an increase in the ratio of currency to checkable deposits. Feige (1980), on the other hand, hypothesizes that activity in the underground economy is likely to be recorded in measures of total transactions but excluded from recorded income. Thus, changes in the ratio of transactions to income are evidence of changes in the relative size of the underground economy. These two "monetary statistics approaches" can be described as the currency-ratio method and the transactions-ratio method, respectively.

This chapter presents estimates of underground activity based on these approaches and some extensions; it points out advantages and potential drawbacks associated with each. In addition, the chapter also examines some of the reasons for the growth of per capita currency holdings, particularly in the form of larger denominations – another observation cited as evidence of underground activity.

Simple currency-ratio method

The first approach to estimating underground economic activity using monetary statistics is based on movements in the ratio of currency to checkable deposits – more simply, the currency ratio.[2] Three assump-

[2] The method was originally suggested by Cagan (1958) to evaluate the upward movements in the currency ratio in World War II. The method was later adopted by Gutmann (1977). The initial estimates of underground GNP made by Gutmann and by Feige covered a period when the levels of deposits in other checkable accounts such as ATS, NOW, and Super NOW accounts were small; they thus ignored these accounts in their work and used the ratio of currency to demand deposits. In the last few years these new accounts have grown rapidly and have tended to substitute for demand deposits rather than for currency; as a consequence, the ratio of currency to demand deposits has risen for reasons totally unrelated to underground activity. Thus, in this chapter, the currency-ratio estimates are based on the ratio of currency to checkable deposits.

Table 5.1. *Computed underground GNP using alternative methods for selected years*[a] *(in billions)*

Year	Simple currency ratio	Modified currency ratio	Econometric model of currency to M2: TW	Econometric model of currency to M2: T	Transactions, 1939 base	Transactions, 1964 base[b]
1950	15.9	21.5	14.5	9.4	27.6	43.1
1955	14.7	15.6	12.8	10.9	1.7	21.6
1960	17.3	17.1	20.7	13.2	−3.4	21.5
1965	31.6	38.6	26.3	17.1	9.6	44.3
1970	62.4	88.6	45.6	25.3	101.0	155.2
1975	150.8	246.0	77.0	46.6	467.3	567.1
1978	226.1	460.2	114.2	80.9	551.1	685.6
1979	317.8	558.5	130.7	88.6	628.4	779.2
1980	372.8	666.9	159.9	116.9	1095.6	1280.1
1981	427.1	767.6	n.a.	n.a.	1765.6	1999.2
1982	449.7	810.5	n.a.	n.a.	n.a.	n.a.
			As ratio to record GNP (%)			
1950	5.6	7.5	5.1	3.3	9.6	15.1
1955	3.7	3.9	3.2	2.7	0.4	5.4
1960	3.4	3.4	4.1	2.6	−0.7	4.2
1965	4.6	5.6	3.8	2.5	1.4	6.4
1970	6.3	8.9	4.6	2.6	10.2	15.6
1975	9.7	15.9	5.0	3.0	30.2	36.6
1978	12.3	21.3	5.3	3.7	25.5	31.7
1979	13.1	23.1	5.4	3.7	26.0	32.2
1980	14.2	25.3	6.1	4.4	41.6	48.6
1981	14.5	26.0	n.a.	n.a.	59.8	67.7
1982	14.6	26.4	n.a.	n.a.	n.a.	n.a.

[a] For a description of each method see the text.
[b] In 1964 it is assumed that underground GNP equals 5 percent of observed GNP.

tions underlie this technique: (1) All underground transactions involve currency exclusively; (2) above-ground activity has a currency ratio that is constant over time; and (3) the underground income velocity of underground currency (i.e., the underground income supported by a dollar of underground currency) is the same as the above-ground currency holdings in that year. The estimated size of underground economic activity can then be derived as the product of underground currency (actual currency less that held in the above-ground sector) and the income velocity of above-ground M1. Table 5.1 lists the resulting estimates of underground gross national product (GNP) under the assumption that the benchmark currency ratio is 0.217.[3] The estimated size of the underground economy grows over time but remains roughly constant as a percentage of recorded GNP until the 1970's; that proportion then increases sharply, reaching a sizable 14.6 percent in 1982.

General currency-ratio method

Another monetary method that was subsequently developed by Feige (1980) generalizes the currency-ratio method. Feige argues that some firms and households use checks in underground transactions because they perceive that the ease of using checks outweighs the costs of leaving a "paper" audit trail; and to the extent that activity in the underground sector is service oriented, income velocity (the ratio of income to money holdings) may be higher in this sector than in the above-ground economy because fewer intermediate transactions occur in producing services. Specifically, Feige makes the following assumptions: (1) The currency ratio in the underground sector is 2; that is, for every two dollars underground participants hold in currency, they hold one dollar in demand deposit balances; (2) the underground income velocity of underground M1 (the sum of currency and checkable deposits) is 10 percent higher than its above-ground counterpart; and (3) in 1964, the base year, underground GNP equaled 5 percent of recorded GNP.[4] The modified currency-ratio estimates of underground GNP for selected years are shown, based on annual averages of the relevant data, in the second column of Table 5.1. In the mid-1960's, this currency-ratio method gives higher estimates of underground GNP than does Gutmann's simple currency-ratio method; beginning in the 1970's the gap between the two estimates widens greatly, and by 1982 the modified currency-ratio estimate of underground GNP, at 26.4 percent of above-ground GNP, is almost twice the estimate derived from the simple currency-ratio approach.

A further variant of the currency-ratio method: Tanzi's model

Another variant of the currency-ratio method (Tanzi, 1983) can be used to estimate underground activity. Tanzi explicitly incorporates the effects

[3] This is the value that Gutmann (1977) estimated for the 1937–41 period; it was assumed that the underground economy did not exist at this time because tax evasion incentives were limited.

[4] For a discussion of the evidence supporting these assumptions, see Feige (1980). In the first chapter of this volume Feige examines the sensitivity of the general currency-ratio model to alternative parametric specifications.

of taxation on the currency ratio. He assumes that the demand for currency relative to M2 rises whenever real per capita income or the rate of interest on time deposits (which are included in M2) falls. The share of wages and salaries in national income, also included in the model, is assumed to have a positive effect on the ratio of currency to M2 to reflect changes in payment practices that have grown to involve greater use of checks. The ratio of currency to M2 is also assumed to be positively related to taxes. The latter assumption reflects the presumed pecuniary advantage of engaging in underground activity as taxes increase and there is an associated induced increase in the demand for currency for underground transactions relative to the other components of M2.

The empirical implementation of the model uses two alternative tax measures: a weighted average tax rate on interest income (TW) and the ratio of total net tax payments to adjusted gross income (T). The model (detailed in Appendix A) is estimated from annual data for the years 1930–80 using the pre-1981 definition of M2. Tanzi defines currency associated with underground activity as the difference between the model's predicted value of currency using the historical values of all explanatory variables (including taxes) and the predicted value if taxes were held constant at zero (i.e., if there were no taxes). As in the simple currency-ratio method, he assumes that the income velocities of underground and above-ground money balances are identical; underground GNP is then the product of underground currency balances and the above-ground income velocity of above-ground M1 balances. Table 5.1 presents the estimated size of underground activity based on the two different tax measures. In sharp contrast to the previous estimates, both of the estimates derived from the tax-driven model tend to stay in a relatively narrow range, around 5 percent of recorded GNP.

The transactions-ratio method

Feige (1979, 1980) also developed an alternative monetary statistics method of estimating underground activity based on the ratio of total monetary transactions to GNP. Instead of using *stocks* of currency and checkable balances, Feige focused on the *flow* of monetary services provided by the stock of M1, namely, the total dollar value of transactions in M1 balances.[5] The key assumption in this approach is that

[5] More precisely, Feige estimates total transactions on the basis of estimated transactions in currency and checkable deposits but chooses to omit transactions in traveler's checks since they must be purchased with either currency or checks.

total transcations are proportional to total economic activity ("total" here means the sum of above-ground and underground activity). Within this framework, transactions can be broken down into three components involving the production of final output, the exchange of existing real or financial assets, and direct transfer payments. Feige recognized that transfer payments exhibit a changing pattern over time and that purely financial transactions associated with asset exchanges probably have increased dramatically in response to various financial innovations. Thus, to derive a transactions measure appropriate for estimating underground activity, Feige deducted a number of major financial transactions and direct transfers from gross transactions to arrive at a net transaction measure; the theory is then reformulated in terms of the proportionality between net transactions and total income.

Table 5.2 depicts the way in which a net transactions series is constructed from the various underlying financial and nonfinancial series. Gross transactions in the table are the sum of estimated currency transactions and total debits to checkable deposits.[6] Three types of financial transactions are then subtracted from this measure of gross transactions: estimated debits to demand deposits for cash withdrawals and withdrawals to other checkable deposits; debits to demand deposits for the purchase of various money market instruments (repurchase agreements, overnight Eurodollars, time and savings deposits, and money market funds); and estimated transactions in the stock and bond market. Finally, several additional adjustments are made to make the net transactions and income series comparable.

[6] We are indebted to Professor Feige for providing these estimates. Feige (1979) initially used Laurent's (1970) method to estimate currency turnover. To illustrate the method, consider a representative economy with $100 in total currency. Every year $10 worth of currency is replaced (redeemed) by the Treasury because it is unfit to circulate any longer. Under these conditions, the average circulation of a given bill is ten years. Laurent assumed that each bill that was redeemed underwent G lifetime transactions while each bill that was still in circulation had undergone only $\frac{1}{2}G$ transactions. Using differential redemption rates by denomination, he computed the cumulative number of times each denomination was used in a transaction and the associated total cash payment per year as a function G. He then chose the value of G that maximized the correlation between total transactions (demand deposits plus currency transactions) and nominal GNP over the period 1861–1967.

In applying Laurent's method to the post-war period, Feige discovered that the transactions velocity of currency declined sharply, from 60 turnovers per year in 1940 to only 17 per year in 1944, principally because of lowering the quality of the currency in circulation to conserve labor and material during the war. Following Irving Fisher's suggestion that currency use patterns change slowly, Feige fit a simple nonlinear time trend to Laurent's data. Groups of large negative or positive residuals from this regression were then matched with archival records indicating administrative decisions to alter the quality of the currency. The fitted values from the regression formed the basis for the turnover estimates used in this method (Feige, 1980, pp. 26–29).

Table 5.2. *Consolidated data underlying transactions approach for selected years (billions of dollars of transactions, annual averages)*[a]

	Gross transactions		Adjustments to gross transactions						Net transactions[j]
Period	Currency[b] (A)	Checkable deposits[c] (B)	Debits to demand deposits arising from cash withdrawals and transfers to other checkable deposits[d] (C)	Other financial transfers[e] (D)	Stock and bond transactions[f] (E)	Foreign transactions[g] (F)	Government payments[h] (G)	Taxes[i] (H)	(I)
1939	396.3	443.2	198.1	41.8	13.0	4.8	11.2	1.8	591.3
1975	4127.4	13747.6	2068.5	427.6	167.0	180.7	353.6	176.3	13112.3
1980	6639.1	31406.6	3522.9	9524.9	523.0	449.5	557.9	346.2	24237.1
1981	7168.8	40858.5	4416.5	12099.1	533.0	507.3	621.6	402.7	30690.3

[a] Data underlying table compiled and adjusted from various underlying sources by Professor Edgar Feige, Department of Economics, University of Wisconsin, Madison.

[b] Based on *estimated* currency turnover rate using Laurent's method as adjusted by nonlinear time trend regression for purported changes in currency quality.

[c] The sum of other checkable and demand deposit transactions. Demand deposit transactions based on turnover concept exclusive of selected financial centers series spliced together between current reporting basis and various earlier concepts; turnover rate for other checkable deposits is set to equal its value in 1978 for 1963–77.

[d] *Estimated* for currency and demand deposits.

[e] Sum of *estimated* debits to demand deposits due to transactions in repurchase agreements, overnight Eurodollars, time and savings deposits, and money market funds.

[f] *Source:* Statistical Abstract of the United States, various years.

[g] Sum of imports and adjusted capital outflows.

[h] Sum of federal transfer payments and wages and salaries of government workers.

[i] Sum of personal contributions to social security and personal income taxes.

[j] Sum of columns A, B, and G less sum of columns C, D, E, F, and H.

Given the net transactions series, the calculation of underground GNP proceeds in much the same fashion as in the currency-ratio method. The proportionality hypothesis between adjusted transactions and income is invoked to estimate underground GNP: above-ground transactions are determined by multiplying the ratio of transactions to GNP in the benchmark period (which is assumed free of underground activity) times recorded GNP in some year. The excess of actual transactions over above-ground transactions in that year represents underground transactions; lastly, applying the benchmark ratio of transactions to income to the underground sector, the underground income supported by the estimated underground transactions can be inferred.

In addition, Feige argues that the above-ground service sector requires fewer transactions per unit of output than do sectors that use more intermediate inputs. Because the service sector has grown relative to the rest of the economy, he expects that in the absence of an underground sector, the ratio of net transactions to income would tend to decline. He argues that basing an estimate of underground GNP on the constancy of this ratio would likely result in understatements of such activity.

Table 5.1 lists alternative transactions-ratio estimates of underground GNP. The estimates in column 5 assume that there were no underground transactions in a 1939 base period whereas those in column 6 use a 1964 base period and the assumption that underground GNP equaled 5 percent of observed GNP in that year. The transactions-ratio estimates of the size of underground activity are even larger than those estimated from the currency-ratio methods, rising from approximately 10 or 15 percent of reported GNP in 1970 to much higher levels in recent years: by 1981 underground GNP is estimated to have equaled more than 60 percent of recorded GNP.[7]

Implied income velocity estimates for the monetary statistics methods

Except for those derived from Tanzi's model, all the estimates based on monetary statistics yield, since the late 1960's, increasing ratios of underground GNP to above-ground GNP, with the acceleration in these

[7] Editor's note: In the transaction method estimates presented in Table 5.1, Porter and Bayer inadvertently added government expenditures to total transactions rather than subtracted them from income. The effect of this error is to raise the estimates of unrecorded income. A corrected series of estimates employing the 1939 benchmark is displayed in Chapter 1, Figure 1.6.

Table 5.3. *Implied total income velocity of money using alternative methods to estimate underground activity and recorded velocity*[a]

Year	Alternative currency-ratio methods: Simple currency ratio	Modified currency ratio	Transactions-ratio method: 1939 base	1964 base	Recorded M1
1950	2.700	2.750	2.804	2.943	—
1955	3.144	3.151	3.045	3.196	—
1960	3.705	3.704	3.559	3.745	3.583
1965	4.378	4.420	4.245	4.455	4.186
1970	4.996	5.120	5.180	5.436	4.701
1975	5.967	6.301	7.077	7.428	5.436
1978	6.929	7.483	7.742	8.125	6.168
1979	7.242	7.879	8.064	8.463	6.400
1980	7.487	8.220	9.288	9.748	6.558
1981	7.864	8.656	10.977	11.520	6.870
1982	7.691	8.479	n.a.	n.a.	6.711
			Average annual growth of implied velocity		
1950–70	3.1	3.4	3.3	3.1	3.5
1975–81	4.7	5.4	7.6	7.6	4.0
1975–82	3.7	4.3	n.a.	n.a.	3.1

[a] Velocity is measured as ratio of sum of above-ground or recorded GNP plus underground GNP to M1 measure.

ratios becoming particularly evident since 1975.[8] For example, the rate of growth of underground GNP derived from the modified currency-ratio approach increased at an 18.6 percent annual rate from 1975 to 1982, almost twice as fast as the above-ground GNP growth rate. An implication of this acceleration in the growth rates of underground activity is a sharp increase in the total income velocity of M1.

Table 5.3 displays the implied level of total GNP velocity – the ratio of the sum of above-ground and underground GNP to the level of M1 – for the alternative monetary statistics methods for the same years reported in Table 5.1; the lower panel displays velocity growth rates over selected periods. As the table indicates, both implied total and recorded

[8] However, as described in what follows, Tanzi's model produces sizable estimates of underground activity when the equation is simulated dynamically rather than statically.

velocity grew about 3 percent from 1950 to 1970. From 1975 onward, the estimated growth rate of implied total income velocity accelerated relative to that of recorded income velocity, with the latter staying close to its long-run historical trends. Consider, for example, the implied total velocity of the transactions-ratio method with a 1939 base period, given in the third column of Table 5.3. From 1950 to 1970 its implied total income velocity grew at a 3.3 percent annual rate; then this growth more than doubled to 7.6 percent per year from 1975 to 1981. That is, taking this estimate of total GNP at face value, it implies that growth in velocity as measured by the ratio of recorded GNP to M1 considerably understates true velocity growth. If sharp changes in velocity over short time intervals are considered unlikely, the estimates of underground economic activity in Table 5.1 are also called into question.

An econometric evaluation of the currency-ratio method

Whereas the underground economy may influence the currency ratio, other, probably more important factors clearly are omitted from consideration by the users of this approach. The conventional macroeconomic approach to analyzing these ratios involves a model based on either an above-ground transactions or an above-ground portfolio theory of the demand for money. The behavior of currency relative to checkable deposits or to M2 can, in fact, be explained with some degree of accuracy by standard econometric demand equations without reference to underground economic activity. Figures 5.1 and 5.2 display the actual and predicted values of the alternative ratios from simulations using the Board staff's quarterly econometric model; the simulations start in the third quarter of 1974 and extend through the third quarter of 1983.[9]

The present demand equations for these components are estimated over various sample periods, all of which ended in the last quarter of 1981. Thus, only the last seven quarters of the simulations represent the out-of-sample period. Accordingly, the close correspondence between the actual and simulated values of these ratios over the entire period does not represent a strong test of the explanatory power of the board model. Over much of this period, particularly the two-and-one-half-year period from 1974, third quarter, to 1976, fourth quarter, checkable deposits – in particular the demand deposit component of checkable

[9] In these dynamic simulations the determinants of the ratios – interest rates, real income, and so forth – took on their historical values. Appendix A presents a brief explanation of their structure.

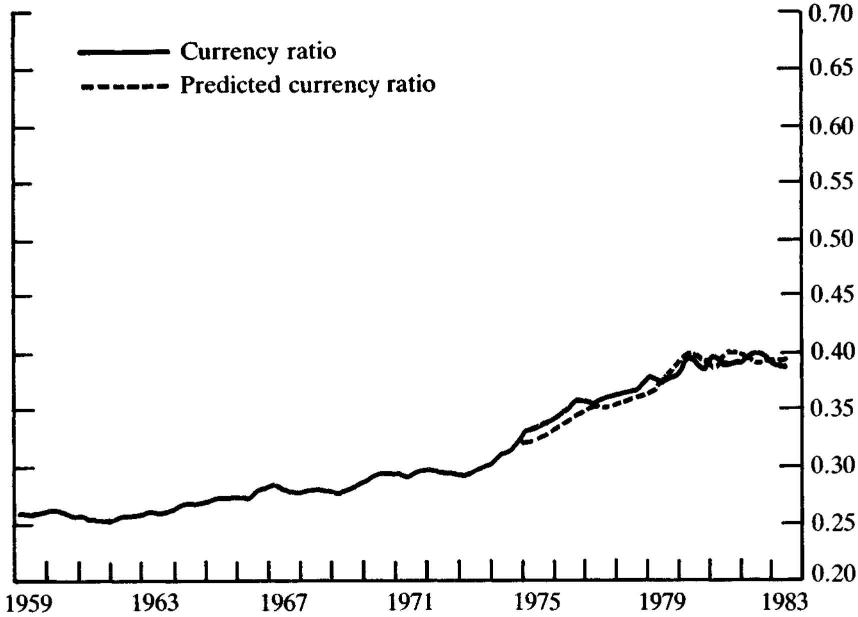

Figure 5.1. Actual and predicted currency ratios.

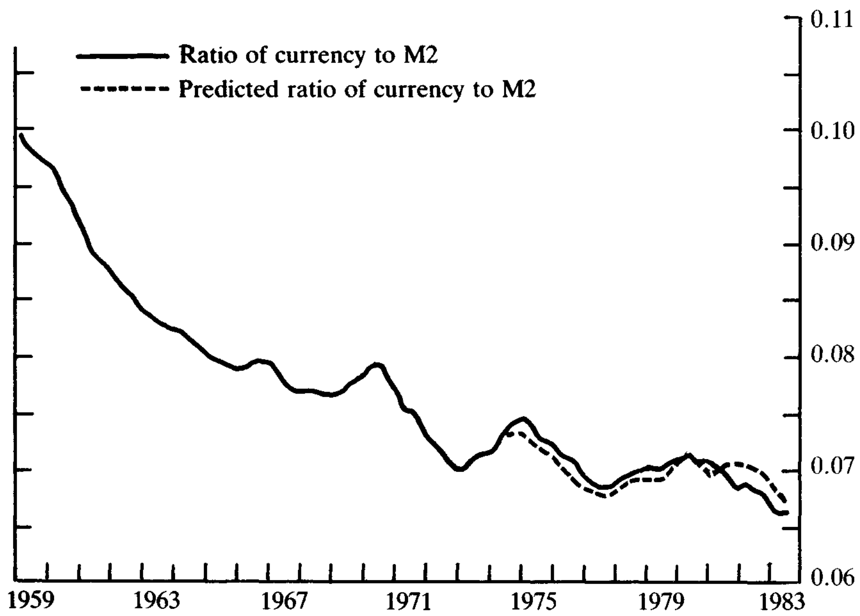

Figure 5.2. Actual and predicted currency-to-M2 ratio.

deposits – grew much less than predicted by a standard equation. To account for this episode, the present equation for demand deposits includes a shift variable (SHIFT) for this period. When it is removed, the model, as do most conventional demand equations, overpredicts demand deposits. The result of this overprediction of demand deposits is an implied underprediction of the ratio of currency to checkable deposits. Some might identify this resulting unexplained spurt in the currency ratio as an argument for the existence of an active underground economy.

Several factors, however, deny this underground economy explanation. First, the board model (Porter and Thurman, 1979) and other models (Garcia, 1978) provide no evidence of unexplained strength in currency itself during this period. Thus, the shortfall in predicting the currency ratio stems principally from the unexplained weakness in demand deposits. Second, this weakness has been examined in depth, and none of the proposed explanations rely on the underground economy.[10] Instead, it appears that in the presence of persistently high opportunity costs of holding demand deposits, deposit holders sought to improve their money management techniques. This quest was aided by improvements in computer and telecommunications technology, by the development of various cash management procedures such as cash concentration accounts and remote disbursement facilities, and by the growing use of new financial instruments that complemented many of these new techniques.

Figure 5.2 indicates that the board money demand models fairly accurately predict the currency-to-M2 ratio. However, since Tanzi's econometric demand model also tracks this ratio fairly accurately, a more detailed analysis of his model is needed.[11] As an alternative to standard models such as the Board model that assume only above-ground transactions and portfolio motives for holding currency and deposits, Tanzi attempts to recognize the presence of the underground economy by including an explicit tax term in his demand equation. The resulting estimates of the size of the underground activity depend importantly on the proper specification and estimation of the tax effect. Taking Tanzi's equation as specified,[12] inspection of the data for the

[10] The mid-1970's episode of demand deposit weakness has been intensively studied by a number of researchers: Judd and Scadding (1982); Porter, Simpson, and Mauskopf (1979); and Enzler, Johnson, and Paulus (1976).

[11] Recall that Tanzi used the older definition of M2.

[12] The Tanzi specification should be viewed as a reduced-form specification. To see this, consider equations for currency holdings of the underground and above-ground sectors. Let t^a be the tax rate on reported income in the above-ground sector and t^u the expected tax rate on underground income. Then in long-run equilibrium, real per

regression reveals that the dominant source of the positive relationship between the ratio of currency to M2 and taxes resides in the data for the period from 1930 to 1945.[13] Indeed, when the estimation period for the regression is restricted to the post-war years 1946–80, the tax variable either enters the equation with the incorrect sign – using T, the ratio of total net tax payments to income – or is not statistically significant – using TW, the weighted average tax rate on interest income (Appendix A). This specification also appears to produce very different estimates of the size of the underground economy depending on how the model is simulated. The Table 5.1 estimates follow Tanzi's work and simulate the equations statically, that is, the simulations are done taking into account the autocorrelation correction term that depends on the actual simulation error made in the previous period. However, a dynamic simulation of Tanzi's model – in which only the error at the beginning of the sample period is taken into account – produces an estimate of underground GNP that is about 18.5 percent of reported GNP in 1980, or about three times larger than that obtained using the static simulation of the equation.[14] Owing to the sensitivity of the estimated tax effects both to the sample period and to the manner in which the simulations are carried out, Tanzi's estimates of the underground economy must be viewed as highly uncertain.

Benchmark, velocity, and recorded GNP assumptions in the currency-ratio methods

The different currency-ratio estimates of the size of the underground economy depend on several critical underlying assumptions. As has been shown, the currency-ratio method produces large estimated increases in

capita currency holdings (C) will be functionally related to real per capita output in the two sectors (y^a and y^u), and the after-tax opportunity cost of holding currency in the two sectors, $r(1 - t^a)$ and $r(1 - t^u)$. Thus,

$$C = C^a[y^a, r(1 - t^a)] + C^u[y^u, r(1 - t^u)]$$

where underground output presumably depends positively on tax rates in the above-ground economy:

$$y^u = f(t^a)$$

Observe that tax rates enter the structural equation in several ways. Changes in enforcement of the tax laws, e.g. that affect t^u, would also alter the relationship between overall currency holdings and the above-ground tax rate.

13 Even for the period before 1946, the specification can be questioned since it does not take into account the introduction of deposit insurance.

14 Feige's (1986c) critique of Tanzi's procedure points out why the dynamic simulation is the appropriate procedure. The results of the dynamic simulation for the post-war years are displayed in Chapter 1, Figure 1.5.

both GNP and implied income velocity during recent periods. However, a small change in assumptions, particularly regarding the magnitude of the currency ratio in the above-ground sector, the so-called benchmark assumption, can drastically change the estimated size of the underground economy. For example, underground GNP can take on negative values whenever the actual currency ratio becomes smaller than the benchmark ratio. The comparable example in Tanzi's model concerns the threshold level of taxes whereby he assumes that underground activity arises as soon as any tax is placed on output. It would perhaps be more plausible to define a normal level of taxation and to associate growth in the underground economy only with tax increases in excess of the normal level. In any case, a more systematic treatment of these benchmarking problems is required.[15] Another important assumption involves the relationship between the GNP velocities of above-ground and of underground money balances. In the modified currency-ratio method the service orientation of the underground sector leads to a higher assumed velocity for this sector relative to the above-ground sector; in the other currency-ratio methods the two velocities are assumed to be equal. The larger negative time trend of demand deposits relative to currency, as has been estimated in the board's quarterly model, suggests that it is more difficult to economize on currency relative to demand deposits. Additional difficulties might also be experienced by underground holders because the transaction costs of converting currency to deposits are larger as a result of the required banking reports associated with large cash deposits or withdrawals. Consequently, quite apart from the service aspect of the underground economy, the income supported by a dollar of underground currency may be lower than the income supported by a dollar in the above-ground sector.

Although the benchmark and velocity assumptions play key roles in the approach, there is a more troublesome assumption that is implicit in the currency-ratio method: Currency-ratio estimates of the ratio of unrecorded GNP to recorded GNP are invariant to the method of estimating recorded GNP. Imagine two different numerical estimates of recorded GNP for a particular year. No matter which estimate was taken as recorded GNP, the currency-ratio estimate of the ratio of unrecorded GNP to recorded GNP would be the same. Thus, it follows

[15] Perhaps they can be related to independent estimates, such as produced by the IRS or the Bureau of Economic Analysis, of the size of the underground economy in various periods – as Feige (1980; Chapter 1 of this volume) has done in some of his papers. However, this solution is not fully satisfactory if these independent estimates are themselves suspect.

that any improvements in the estimate of recorded GNP by the Bureau of Economic Analysis would not change the currency-ratio estimate of the ratio of total GNP (recorded and unrecorded activity) to recorded GNP.

To summarize, without a more definitive treatment of the underlying velocity and benchmark assumptions, it is difficult to assess the final results of the alternative currency-ratio procedures. In addition, the currency-ratio estimates are not sensitive at all to changes in recorded GNP estimates. Presumably, improved estimates of recorded GNP should alter the estimated ratio of unrecorded to recorded GNP, but the available currency-ratio procedures do not allow for this possibility.

An evaluation of the transactions-ratio method

Econometric issues

It is more difficult to provide an econometric evaluation of Feige's transactions-ratio method since there is no established theory of total transactions, as there is for income velocity, which depends on the demand for money. However, casual inspection of the ratio of transactions to income suggests that it has moved positively with interest rates over much of this period. In a recent paper, Porter and Offenbacher (1984) offer a partial explanation for such movements based on an inventory model of money holdings under uncertainty. They show that debits to demand deposits for business firms should be positively related to both interest rates and a scale variable (which serves as a proxy for the size of the firm) and negatively related to the costs of making transactions.[16] Figure A5.1 in Appendix A shows predicted values from this model for the ratio of total demand deposit transactions outside of New York to nominal GNP from this model together with actual values of the ratio. As the figure indicates, some of the major movements in this debits-to-GNP ratio can be explained without reference to factors associated with the underground economy. However, this evidence should be regarded as highly tentative because no theory of total transactions is well established and the simulation results shown in Figure A5.1 are merely within-sample predictions. Until more experience with out-of-sample predictions is obtained, the apparent good fit of the Porter–Offenbacher model should be viewed skeptically.

[16] The particular proxy used for transaction costs is described in Simpson and Porter (1980). Also, for simplicity, the scale variable is taken to be recorded GNP.

Other issues

In comparison with the various currency-ratio methods, the transactions method has several distinct advantages, at least in theory. The method does not require any assumption concerning the relative income velocity in the above-ground and underground sectors. Nor does it require an assumption that currency is the exclusive medium of exchange in the underground sector or that currency and deposits are used in a given ratio in these sectors; it treats currency and deposits in a symmetric fashion. Moreover, improved estimates of recorded GNP can modify the estimate of the ratio of underground GNP to recorded GNP in the proper direction; that is, an increase (decrease) in recorded GNP would lower (raise) this ratio.

On the other hand, this method does require the specification of a transactions ratio in the above-ground sector, and similar to the other methods, the benchmark ratio that is chosen is a critical assumption. In practice, however, data limitations are the single most important problem in implementing the transactions method: Many assumptions must be made to develop the estimates of the necessary data series from existing data sources. As was indicated earlier, measurements of the turnover of the currency stock do not exist, resulting in the need to use an indirect estimation procedure. The problems of estimating currency turnover are relatively minor, however, compared to those associated with netting financial and real asset transactions from gross transactions.[17]

The portion of gross checkable transactions associated with demand deposits is not split into financial and nonfinancial transactions. Currently, the only data directly bearing on such a split exist for major New York City banks.[18] These data represent gross transactions that are almost exclusively financial given that these banks represent many of the nation's leading money center banks. In addition to removing the major New York City banks from his transaction series, Feige also uses an older historical series on debits at selected financial centers to eliminate some additional transactions that are also likely to be dominated by financial movements. But since this older series is no longer collected,

[17] Because there are no reliable data to indicate what portion of the currency stock is held abroad, any inference about domestic currency transactions could be overstated.

[18] In August 1983, demand deposit debits to all insured banks were $111.5 trillion at an annual rate; of this total, $48.4 trillion were transacted at these major New York City banks.

the relationship between it and any current series cannot be benchmarked from the existing data sources. There are also significant problems in netting out demand deposit debits involving the purchase of other financial claims. For example, comprehensive measures of the volume of activity on many types of security markets do not exist, nor are there direct estimates on the turnover rate of several important money market instruments such as repurchase agreements. In addition, improvements in money management techniques, particularly the use of cash concentration accounts by nonfinancial firms, result in debits to demand deposits that are purely financial in nature. These debits are not necessarily eliminated by the netting procedures used in the transactions-ratio method because they involve an increase in transactions among the demand deposit accounts held by nonfinancial corporations at different banks and not the purchase or sale of a money market instrument (Carlson, 1982). Some corporations apparently have been very efficient in reducing their holdings of M1 by using such methods.[19]

In examining the recent estimates from the transactions methods, it appears that increases in the transactions ratio are largely due to checkable deposits, not currency. For example, the estimated ratio of net transactions to recorded GNP increased from 8.46 in 1975 to 10.03 in 1981; but if the proportion of currency transactions was held fixed at its 1975 level, the transactions ratio would still have risen to a value of 9.54. Thus, the major explanation for the increase in the ratio lies in transactions involving deposits, not currency. Since the likelihood of being "caught" is probably higher when checkable deposits rather than currency are used in the underground economy, it would seem counterintuitive to associate all the increase in the income implied by this increase in the transactions ratio with underground transactions. Instead, it is more likely that at least part of the observed increase in the transaction ratio was related to purely financial transactions. This interpretation seems to be confirmed from an examination of the transactions-ratio method's growth of implied GNP velocity from 1980 to 1981 (Table 5.3). The resulting 18.1 percent annual rate of velocity growth in 1982 is about four times faster than recorded velocity growth for that year. Such a large increase appears unlikely and suggests that some financial component of the transactions has not been properly netted out in the recent estimates.

[19] Based on Board's flow-of-funds data, the share of M1 held by nonfinancial businesses fell from 34.2 percent in the first quarter of 1959 to 18.6 percent at the end of the third quarter of 1983, with the largest part of this decline taking place in the 1970's.

Summary of monetary statistics approach

There are a number of estimates of underground activity based on the monetary statistics methods. Nearly all of these estimates imply a relative rise in underground activity and in the total income velocity of money since 1970. The simple and modified currency-ratio estimates depend on a number of tenuous assumptions – the most critical being that (1) the currency ratio, the ratio of currency to checkable deposits, in the above-ground sector is constant despite changes in economic determinants such as interest rates and the own rate of return on NOW/ATS balances; (2) the ratio of underground to above-ground or recorded GNP is invariant to the way in which recorded GNP is measured; and (3) currency and checkable deposits are used in given proportions in the underground sector. The transactions-ratio method avoids these problems but does not appear to produce credible estimates in recent periods, owing to presumed difficulties in separating out purely financial transactions from other transactions.

Finally, there is the explicit econometric model of the ratio of currency to M2, which represents the underground economy indirectly through the use of tax variables. Unlike the other estimates, this method does not indicate any relative increase in the underground sector relative to total economic activity. However, this method does not also appear to be reliable: It makes the same invariance assumption as the other currency-ratio procedures; it does not allow for any use of checks in the underground economy; and it fails to estimate the tax effects very precisely in the post-war period.

An evaluation of the currency data

For many, the most compelling evidence concerning the existence of the underground economy involves the remarkable level of per capita currency holdings. At the end of 1982, currency holdings, including vault cash, stood at $675 per capita with just under 40 percent in $100 bills. These figures seem to contradict everyday experience. However, since there are no reliable estimates to take account of the portion of the currency stock held abroad, the holding of currency by domestic residents is clearly overstated.

Despite the high level of per capita currency balances, Figures 5.2 and 5.3 show that on-balance aggregate currency over the past twenty years has been declining, not rising, relative to M2, traveler's checks, domestic nonfinancial credit, nominal measured GNP, and nominal measured personal consumption expenditures. In the case of M2, this movement should not be altogether unexpected since the nominal rate of return on M2 has moved up sharply over this period as a result of deregulation and higher nominal interest rates while the nominal return on currency has remained at zero. More striking is the fact that a similar declining pattern is apparent, at least through the mid-1970's, for the ratio of currency to traveler's checks even though traveler's checks, similar to currency, bear no nominal rate of return and, unlike currency, leave a paper trail (Figure 5.3). Finally, currency movements over the past years have been highly predictable in models such as the Board model, where no reference to the underground economy is made (Figure 5.4).

Nonetheless, some suggest that the accurate prediction of currency balances by econometric models may be fortuitous. Since currency holdings are the sum of above-ground and underground holdings, a relative decline in currency holdings in the above-ground sector due to changes in payment practices may offset a relative increase in underground currency holdings, thereby leaving the sum unaffected. More frequently, use of credit cards is perhaps one method by which above-ground currency holders may have economized on currency; however, credit card use represents only a small proportion of the estimated volume of total currency transactions – just over 2 percent in 1981 based on the currency transaction estimates in Table 5.2. A second suggested factor in the possible decrease of above-ground currency use is that an increasing fraction of individuals are paid by check rather than with currency. This factor is not accounted for in the standard currency demand specification; however, when this effect is captured with a series similar to the one used by Tanzi, the predictions of the Board currency equation are not materially altered.

On the other hand, it might be argued that since the mid-1950's aggregate currency balances (including vault cash) have only about kept pace with inflation so that real per capita currency holdings have been almost unchanged (Figure 5.5). Thus, if real per capita instead of total currency holdings were used in the monetary statistics approach, the relative size of the underground economy would be approximately unchanged from the early post-war period until now.[20]

As mentioned earlier, those who assert that there is a growing underground economy sometimes point to the rising proportion of $100 bills

[20] Because the total economy has grown over this period, the fact that real per capita currency holdings are relatively constant would imply, other things equal, that the underground economy has declined relative to the above-ground economy. However, the increase in the opportunity cost of holding currency and autonomous improvements in managing currency have apparently offset the increased level of transactions, leaving real per capita currency holdings about unchanged.

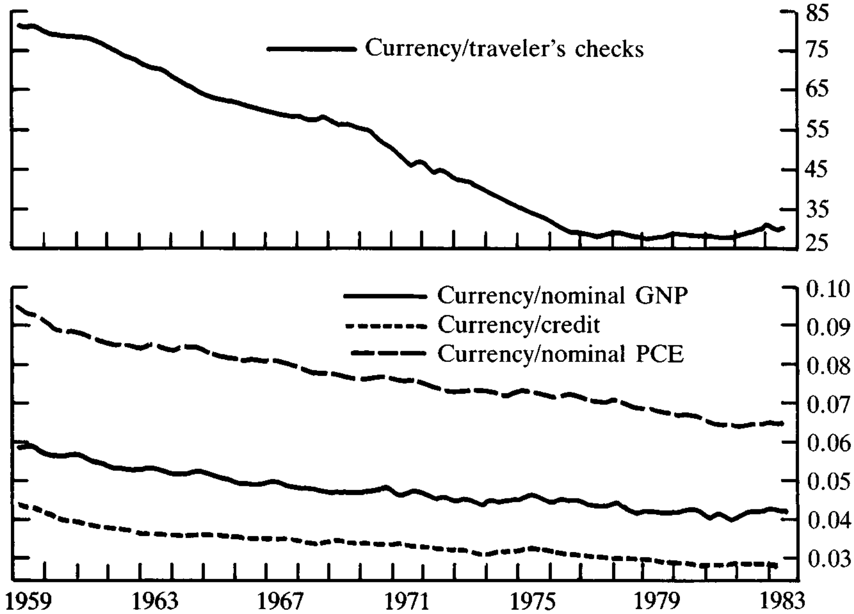

Figure 5.3. Ratios of currency to traveler's checks, GNP, credit, and prices.

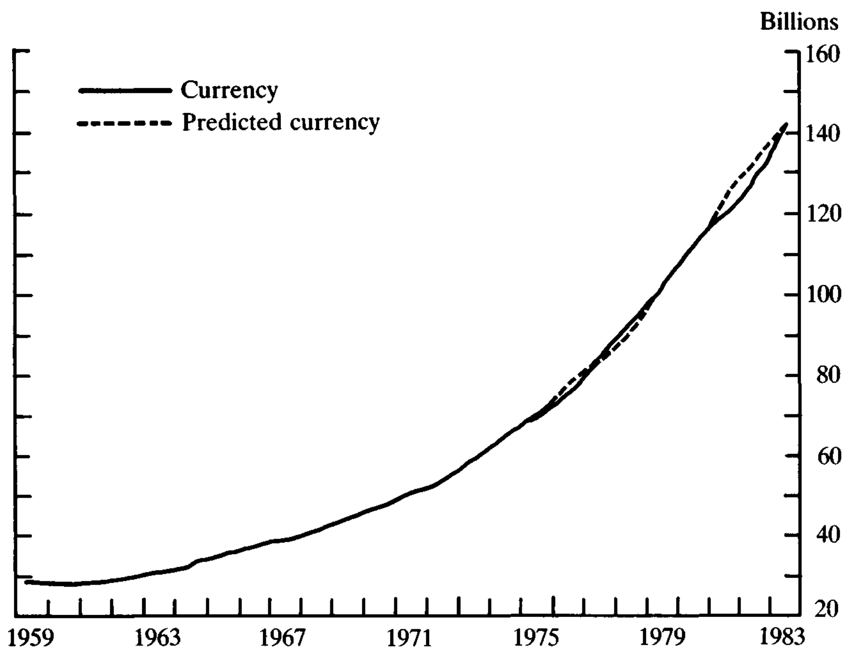

Figure 5.4. Actual and predicted currency holdings.

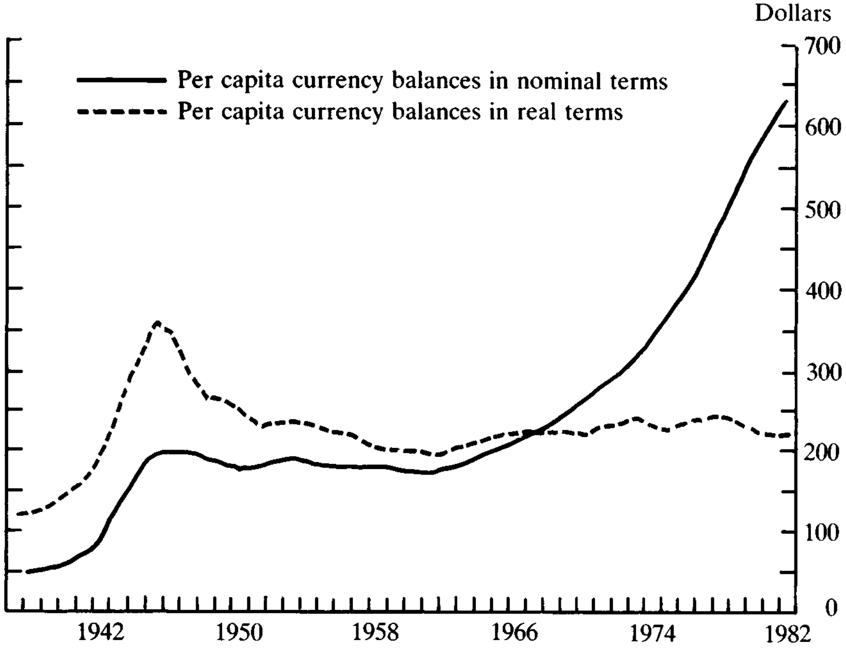

Figure 5.5. Per capita currency balances.

in the currency stock (see Figure 5.6). Even in real terms, the proportion of $100 bills is striking (Figure 5.7). Either this is evidence of increased underground economic activity or it reflects the desired behavior of above-ground transactors.

Focusing on the latter explanation, it should be noted that since 1969 the $100 bill has been the largest denomination issued.[21] Thus, increases in the price level that tend to increase the dollar size of transactions should lead to greater proportionate use of $100 bills, other things equal. This "convenience" aspect of $100 bills is not shared by other denominations. By drawing upon a model recently proposed by Cramer (1983), the precise importance of hundreds in the mix of denominations can be determined. Cramer assumed that economic agents attempt to minimize the number of physical units of currency used in an exchange of a given transaction size. Table 5.4 presents the results of applying Cramer's model to the various bill denominations in the United States for various transaction size ranges.[22] The estimates were constructed

[21] Denominations larger than $100 have not been printed since 1946. The use of these denominations had declined sharply over the years, and in 1969, there appeared to be no need to resume printing of the larger denominations – $500, $1,000, $5,000, and $10,000 bills.

[22] We are indebted to Gary Anderson of the Board staff for his technical assistance in putting together this table.

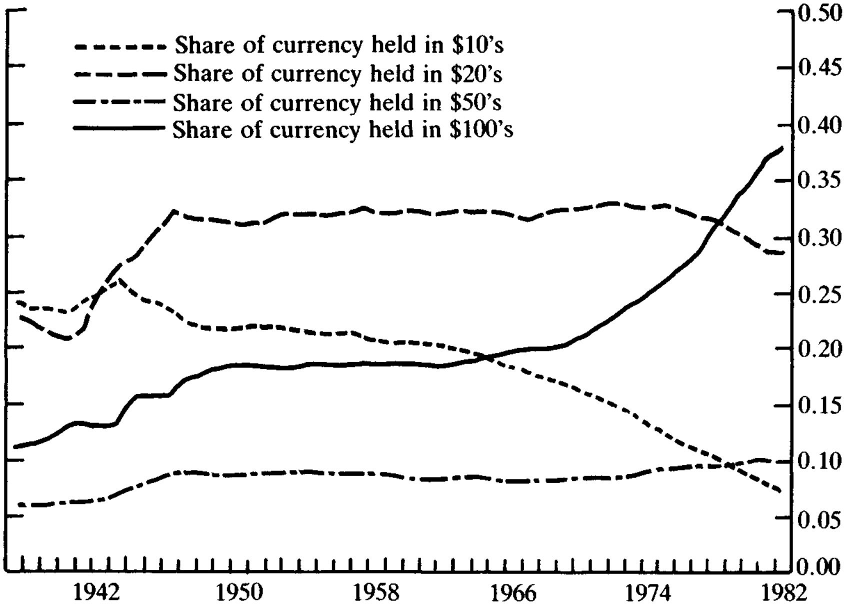

Figure 5.6. Shares of currency held in various denominations.

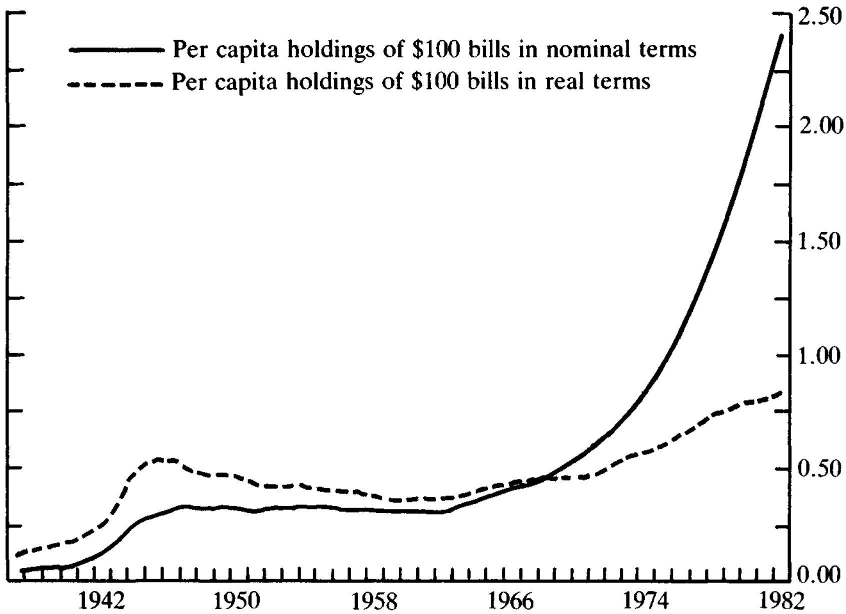

Figure 5.7. Per capita holdings of $100 bills.

Table 5.4. *Relationship between average transaction size and share of currency held in selected denominations*

β Parameter	Average size of transactions ($)	Share of $100 bills in optimal mix of denominations (%)	Share of $50 bills in optimal mix of denominations (%)	Share of $20 bills in optimal mix of denominations (%)
10	12.69	12	13	22
20	25.38	19	20	33
30	38.08	25	28	29
40	50.77	31	32	23
50	63.46	37	34	18
60	76.15	43	32	14
70	88.85	49	29	13
80	101.54	56	25	12
90	114.23	62	21	10
100	126.92	66	19	9

under the assumption that all transactions up to a certain size (β) were equally likely to occur (i.e., followed a uniform distribution) and that transactions larger than that size were assumed to follow a Pareto distribution, so that each successive transaction size beyond β was a little less likely to take place than the preceding transactions size.[23] Although it is difficult to compare the model's predictions with the actual mix of denominations in the United States, the table indicates that as the dollar size of individual transactions increases, the proportion of hundreds in the optimal mix of denominations rises. Thus, for example, as the average transaction goes from a little over $25 ($\beta = 20$) to a little over $100 ($\beta = 80$), the proportion of $100 bills goes from a 19 percent share to a 56 percent share.

From this perspective, changes over time in the share of currency held in various denominations are not too surprising (Figure 5.6). In 1978 the share of currency in $100 bills surpassed the share in $20 bills. The figure

[23] That is, the distribution function for transactions of size x was

$$f(x) = \begin{cases} c & \text{if} \quad x \leqslant \beta \\ c\left(\dfrac{\beta}{x}\right)^{\alpha+1} & \text{if} \quad x \geqslant \beta \end{cases}$$

where $c = \alpha/\beta(\alpha + 1)$ and β is the upper limit of the uniform portion of the distribution. The Pareto parameter α was set equal to 1.65. This is the approximate value estimated for a variant of this model used to explain per capita holdings of $100 bills.

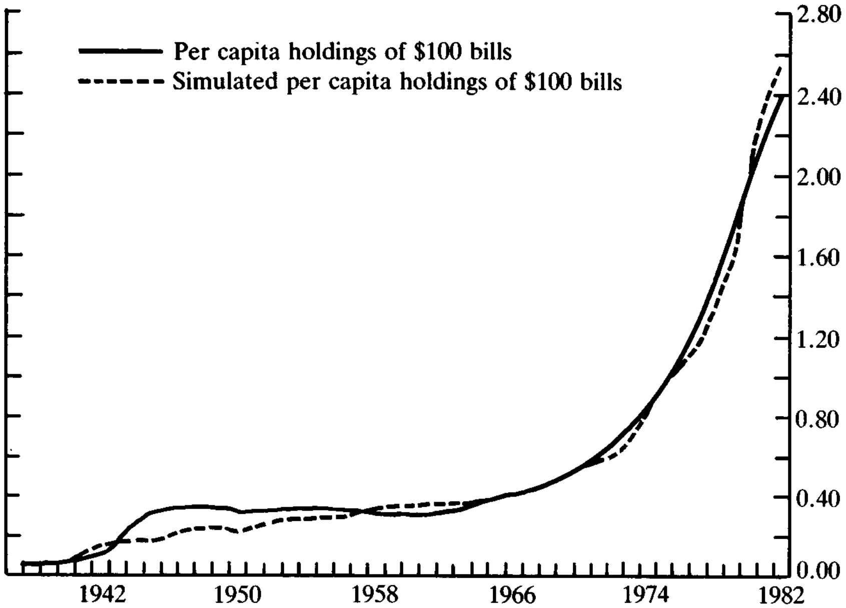

Figure 5.8. Actual and predicted per capita holdings of $100 bills.

shows that the last time a similar event occurred was in 1942, when the amount of money represented by the $20 denomination became larger than the amount held in $10 bills. Over the period from 1942 to 1970, per capita consumption expenditures grew from $605 to $6,048. Thus, whereas per capita consumption expenditures increased by a factor of 10, the size of the denomination in which the largest proportion of currency was outstanding increased only by a factor of 5.

Another way of evaluating the proposition that the growth of $100 bills may have an above-ground explanation is shown by the model developed in Appendix B, which focuses on hundreds exclusively. The model is derived from the following assumptions: (1) For transactions above a given size, the distribution of transactions follows a Pareto distribution; (2) over time, changes in the price level shift the size distribution proportionally; and (3) the number of $100 bills an individual holds is proportional to the probability that he or she will enter a transaction that involves at least a certain size. Under these assumptions, the model explains per capita holdings of $100 bills as a function of the price level. As Figure 5.8 indicates, the regression equation implied by the model performs quite adequately in the out-of-sample period, explaining a substantial part of the recent increase of per capita holdings of $100 bills. Thus, these theoretical and empirical results suggest that the growth of $100 bills may principally be related to economic and institutional forces at work in the above-ground economy.

Although the amount and form of currency holdings may appear suspect at first glance, the increase of per capita currency and $100 bill holdings can be explained without reference to underground activity. Increases in the price level as well as the available denominations of currency can account for patterns of currency holdings.

Conclusions

The analysis of monetary statistics has not progressed to the point where it can provide reliable estimates of underground economic activity. This data source does not provide firm support for the hypothesis that the share of the underground economy in the total U.S. economy has grown over time.

At present, the currency-ratio and modified currency-ratio methods rely on assumptions made solely for technical convenience rather than for consistency with either underlying economic theory or other empirical regularities. The presumed constancy of the currency ratio in the above-ground economy, despite ongoing changes in important economic determinants such as interest rates, is an assumption of convenience that underlies this approach. Although Tanzi has attempted to address this problem through the use of taxes in an explicit regression model, his work does not resolve the size of the estimated tax effects. All of these currency approaches implicitly make the questionable assumption that the ratio of above-ground GNP to recorded GNP is invariant to changes in the estimate of recorded GNP. The transactions-ratio method, on the other hand, treats checkable deposits and currency symmetrically and also avoids the invariance assumption of the various currency-ratio methods. Because it is hard to separate out purely financial transactions from gross transactions, however, implementing this technique in the United States is exceedingly difficult with present data sources. Perhaps as a result of this problem, the transactions-ratio estimates of underground activity for recent periods seem too high.

Despite these problems, the issues raised in evaluating underground economic activity pose some challenging questions regarding the use of currency and deposits as transactions media in the total economy. Perhaps as more satisfactory data sources and methodology are created, better answers to these questions can be found.

APPENDIX A

Empirical financial equations

Equations in the Federal Reserve Board (FRB) model

The demand equations for the components of M1 – currency (including traveler's checks), demand deposits, and other checkable deposits – as they currently appear in the Board's quarterly econometric model (Brayton and Mauskopf, 1985) are available on request from the author. Briefly, the equations for demand deposits and other checkable deposits are based on the same theoretical transactions model and, therefore, are estimated simultaneously and have similar empirical specifications. The per capita demand for deposits is a function of per capita income and the opportunity cost of holding the deposit, the difference between some market rate of interest and the own rate of deposits. In the case demand deposits, the own rate is zero, whereas for other checkables the own rate is R_{now}, the rate of interest earned on NOW accounts. Also worth noting is the fact that the interest rate term in these equations appears in a nonlinear fashion, thus allowing the interest rate elasticity of demand to change with the level of interest rates.

Mnemonics for equations

C = currency (Tanzi)
GNP = nominal GNP
N = population
PCE = personal consumption deflator (1972 = 100)
R = rate of interest paid on time deposits
RATCHET = cash management ratchet
RFF = federal funds rate
T = ratio of total income tax payments to income (Tanzi)
TW = weighted average tax rate (Tanzi)
U = error term for autocorrelation correction
WS/NI = ratio of wages and salaries in national income (Tanzi)
Y = real per capita income (Tanzi)

Tanzi's estimation results[24]

The sample period is 1930–80. Using the weighted average tax rate,

[24] Tanzi's equations were corrected for serial correlation with a first-order Cochrane–Orcutt correction. The values of these rhos, however, were not available.

United States

$$\ln C/\text{M2} = \underset{[-3.61]}{-5.0262} + \underset{[5.81]}{0.2479}\ln(1 + \text{TW}) + \underset{[5.33]}{1.7303}\ln(\text{WS/NI})$$
$$-0.1554\ln(R) - 0.2026\ln(Y) + \varrho_1 U_{-1} + e$$
$$R^2 = 0.950 \qquad \text{DW} = 1.576$$

Using the average tax rate,

$$\ln C/\text{M2} = \underset{[2.93]}{-4.2005} + \underset{[5.26]}{0.3096}\ln(1 + T) + \underset{[4.76]}{1.5791}\ln(\text{WS/NI})$$
$$\underset{[3.37]}{-0.1603}\ln(R) - \underset{[2.22]}{0.2804}\ln(Y) + \varrho_2 U_{-1} + e$$
$$R^2 = 0.947 \qquad \text{DW} = 1.677$$

Best reproduction of Tanzi's results[25]

The sample period is 1930–80. Using the weighted average tax rate,

$$\ln C/\text{M2} = \underset{[-3.60]}{-5.0276} + \underset{[5.78]}{0.24791}\ln(1 + \text{TW}) + \underset{[5.32]}{1.7304}\ln(\text{WS/NI})$$
$$\underset{[-3.66]}{-0.15583}\ln(R) - \underset{[-1.87]}{0.20178}\ln(Y) + \underset{[8.14]}{0.75189}U_{-1} + e$$
$$R^2 = 0.951 \qquad \text{DW} = 1.574$$

Using the average tax rate,

$$\ln C/\text{M2} = \underset{[-2.93]}{-4.219} + \underset{[5.23]}{0.30913}\ln(1 + \text{T}) + \underset{[4.75]}{1.5827}\ln(\text{WS/NI})$$
$$\underset{[-3.38]}{-0.1611}\ln(R) - \underset{[-2.18]}{0.27712}\ln(Y) + 0.83188U_{-1} + e$$
$$R^2 = 0.948 \quad \text{DW} = 1.680 \quad \text{residual standard error} = 0.046788$$

Post-war estimation results using Tanzi's specification

The sample period is 1946–80. Using the weighted average tax rate,

$$\ln C/\text{M2} = \underset{[-0.45]}{-0.40611} + \underset{[0.29]}{0.018224}\ln(1 + \text{TW}) + \underset{[2.22]}{0.49337}\ln(\text{WS/NI})$$
$$\underset{[-0.48]}{-0.016408}\ln(R) - \underset{[-1.23]}{0.095747}\ln(Y) + 0.92868\,U_{-1} + e$$
$$R^2 = 0.989 \quad \text{DW} = 2.10 \quad \text{residual standard error} = 0.018345$$

Using the average tax rate,

[25] Although Tanzi provided his data and estimation results, we were not able to reproduce his estimates exactly.

$$\ln C/M2 = \underset{[0.19]}{0.1753} - \underset{[-0.84]}{0.049305}\ln(1 + T) + \underset{[2.57]}{.57418}\ln(WS/NI)$$
$$-\underset{[-0.62]}{0.051654}\ln(Y) + \underset{[15.02]}{0.93041}\,U_{-1} + e$$

$R^2 = 0.990$ DW = 1.97 residual standard error = 0.018114

Debits equation

The sample period is 1962:1–1983:2.

$$\ln\frac{\text{debits}}{\text{GNP}} = \underset{[-2.02]}{-0.91548} + \sum_{i=0}^{3} a_i \ln(\text{RFF})_{-i} + \sum_{i=0}^{3} b_i \ln(\text{GNP})_{-i}$$
$$+ \sum_{i=0}^{3} c_i(\text{RATCHET})_{-i} + \underset{[13.16]}{0.8175}\,U_{-1} + e$$

$a_0 = 0.012462$	$b_0 = 0.29977$	$c_0 = 0.0018843$
$a_1 = 0.034339$	$b_1 = 0.073013$	$c_1 = 0.0023097$
$a_2 = 0.032759$	$b_2 = 0.016727$	$c_2 = 0.001296$
$a_3 = 0.0077217$	$b_3 = 0.030554$	$c_3 = 0.0089329$
$\Sigma a_i = 0.087281$	$\Sigma b_i = 0.38661$	$\Sigma c_i = 0.006035$
[2.15]	[5.26]	[3.13]

$R^2 = 0.99613$ DW = 2.30 residual standard error = 0.020932

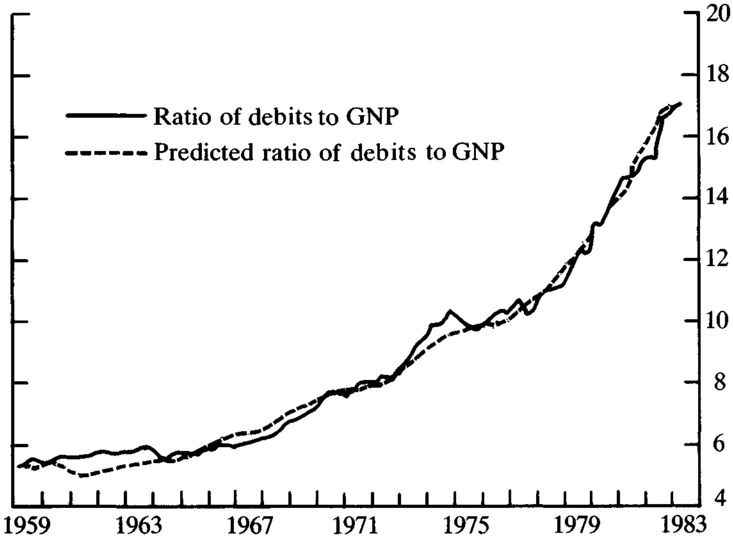

Figure A5.1. Actual and predicted transaction ratios.

APPENDIX B: ASSUMPTIONS FOR MODEL OF \$100 NOTES

1. In 1967, when the consumer price index (CPI) was 1, the distribution of the sizes of currency transactions that were larger than some dollar amount a_0 was Pareto with parameter α:

$$f_0(x|x > a_0) = \alpha a_0^{\alpha}/x^{\alpha+1} \qquad x > a_0$$

2. The transactions that took place in later years were identical to those that occurred in 1967 except that each one involved P_t times as much money as its 1967 counterpart, where P_i is the CPI index in year t.

3. The number n_t of \$100 bills a person holds in year t is proportional to the probability he will enter a transaction that involves at least τ dollars, say, $n_t = k\Pr(x_t > \tau)$.

4. The measured number of \$100 bills per person is the product of n_t and u_t, where $\ln u_t$ has mean 0 and variance σ^2:

$$\tilde{n}_t = n_t u_t$$

Given these assumptions, it is easy to show that (1) the probability that a transaction exceeds τ is proportional to the conditional probability that it exceeds τ given that it exceeds that year's threshold $P_t a_0$, (2) transactions in later years that exceed $P_t a_0$ are also Pareto distributed with parameter α; and (3)

$$\ln n_t = \alpha \ln(kca_0/\tau) + \alpha \ln P_t + \ln u_t$$

It follows that we can estimate α by regressing the logarithm of per capita holdings of \$100, $\ln n$, on the logarithm of the price level, but we cannot distinguish among the remaining parameters of the model, k, c, a_0, and τ. Using a serially correlated error structure, estimates of this equation are (sample period, 1955–75)

$$\begin{aligned} \ln n = {} & \underset{[-41.75]}{0.83864} + \underset{[18.67]}{1.6797} \ln(\text{CPI}) \\ & + \underset{[9.97]}{1.4858}\, U_{-1} - \underset{[-5.07]}{0.69338}\, U_{-2} + e \end{aligned}$$

$R^2 = 0.997$ DW = 1.80 residual standard error = 0.01896