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Complexity

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Aristotle and developed by Tarski. Within a coherence truth-theory, a proposition φ is true if and only if it is consistent with a given set Γ of propositions, that is, if no contradiction can be derived in case we add φ to Γ.

In Bayesian probability theory, an assignment of degrees of belief is said to be coherent if and only if it is not susceptible to Dutch book, that is, if and only if betting according to these degrees of belief does not open the bettor up to the possibility of loss whatever the outcome. *See also* CONSISTENCY, PROBABILITY, INTERPRETATION OF; PROPOSITION; SET; TRUTH; ARISTOTLE; TARSKI, ALFRED. [FB]

Completeness. *See* THEOREMS.

Complexity. There is no single, agreed upon definition of what it is to be complex, but rather a cluster of related notions covering both epistemological and ontological aspects of complexity. Of those most relevant to logic are definitions of algorithmic complexity arising from information theory, and applied to strings in some specified formal language. The best-established of this class of definitions is *Kolmogorov complexity* (KC). The KC of a string of binary digits is measured by the length of its shortest description. Thus the string '101010101010101010101010' can be (fully) described as '12 repetitions of "01" ', whereas the most efficient way to describe a disordered string such as '011000101011101101100010' may be to write down the entire string. One implication of the KC measure is that random strings have the highest complexity. They are also *incompressible* in the sense that there is no way of providing a specification of a random string that is shorter than the string itself. To make the KC measure precise, it must be relativized to a particular (formal) language of description. A quite separate notion of complexity in logic, sometimes known as *quantifier complexity*, measures the complexity of propositions in predicate logic based on the number of alternating blocks of quantifiers occurring in the proposition. *See also* LOGIC, PREDICATE; PROPOSITION; QUANTIFIER. [ABa]

Computability. A function is computable when it is calculable in a definite (i.e., algorithmic) way in a finite number of steps. A set is computable when its characteristic function (the function f such that $f(x)=1$ when x belongs to the set) is computable. Several precise definitions have been proposed in order to explicate this somewhat vague notion, as a result of the works of Church, Gödel, Kleene and Turing. Computable functions have been defined