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Inverting Kite Structure & Stability for Airborne Wind Energy

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Airborne Wind Energy (AWE) is an emerging method of extracting clean energy from high altitude airstreams with tethered kites, but inflated kites presently lack actuators to control lift and roll. Inverting wings show promise at providing both forms of maneuverability and have so far been researched to validate lift vector manipulation. This capstone project aims to demonstrate the applicability of inverting wings to roll control by segmenting an inflated wing into sections with independent inversion control, inducing a roll moment. The preliminary results from this novel wing design suggest that double inversion does indeed facilitate roll control, but further roll analysis is required using automated inversion torque motors. All other structural, functional, weight savings, and cost reduction designs in the inverting wing, its wing mount setup, and the force-moment measurement system performed nominally.

1. Introduction

As the effects of climate change present unparalleled dangers to humankind, diverse sources of renewable energy are necessary to move away from carbon-emitting fuels. Winds near the surface of the Earth contain nearly one hundred times the amount of energy required of the world power demand, but less than a quarter of that energy can be reached by modern wind turbines [1]. At higher altitudes, airstreams speed up dramatically, increasing the energy density eight-fold for every double in wind velocity. Airborne wind energy (AWE) is an emerging field that aims to harvest wind energy at these higher altitudes with aerial platforms—generally referred to as kites—tethered to electric generators on the ground. As a kite produces lift and ascends, it unwinds a spool of tether coupled to its generator. The spinning generator thus creates electricity, and after reaching a designated apogee, the kite is reeled back to a lower altitude [1]. With substantial reductions in infrastructure, investment, and total weight compared to conventional wind turbines, and with the subsequent benefits of heightened mobility, greater energy availability, and the possibility of anchored wind farms in currently inaccessible deep waters, AWE has the potential to spread clean energy production nearly anywhere in the world [2,3].

At present, most AWE research related to energy efficiency focuses on flight pattern optimization of the ascending phase [2]. In contrast, existing literature for soft-bodied kites scarcely attempts to minimize the energy expended during the reel-in period. These lighter and consequentially safer kites have been restricted to energy-intensive reeled descents because existing aerodynamic actuators are too heavy for their fabric wings [1,4]. Since early 2021, a novel lift control mechanism designed for inflated wings has been investigated in the Department of Engineering at Swarthmore College to allow AWE kites to completely reverse their direction of lift. Named “inverting airfoils”, inversion occurs when the circular leading edge of an inflated airfoil rotates to pull in fabric from one surface and pay it out to the other, effectively morphing the airfoil’s camber. As demonstrated in Fig. 1.1, complete inversion of an airfoil mirrors the original camber over the chord line, and additional interior fabric supports constrain the original and final airfoil shape. On AWE kites, inverting wings could allow the aircraft to controllably power dive to minimize time and energy lost in descent while requiring little additional weight or mechanical complexity. Additionally, AWE kites with inversion occurring independently at multiple points along its wings may be able to roll from the unequal moments induced by changing lift vectors. Such an inverting roll control could help inflated AWE kites navigate their complex flight maneuvers without using multiple tether lines from the ground.

The purpose of this senior design project is to investigate the prospect of using multiple inversion segments along a wing to induce a moment on the wing around its roll axis. Inversion perfectly mirrors an airfoil’s lift vector over its chord line, so applying inversion to roll control may also facilitate aerodynamic banking without producing adverse yaw [5]. Such a roll method has never been developed according to the author’s knowledge and would not only bring lightweight flight control to AWE kites but also a simplified roll model to aerodynamic theory.

Fig. 1.1 Inversion of an airfoil occurs when the circular leading edge (red) of an inflated fabric airfoil (black) turns to pull in fabric from one side and pay it out to the other, shown in three successive snapshots. Not pictured are the internal fabric supports that maintain the airfoil’s shape in positive and negative inversion.
2. Prior Research

In the summer of 2021, inverting wings were first proposed by the author through research in the Department of Engineering at Swarthmore College. Inverting airfoils, or the cross-sections of inverting wings, were subsequently studied through experimental testing, computational optimization, and testing equipment construction. A summary of each major advancement to the development of inverting airfoils is presented.

2.1 Inverting Airfoil Selection

Inverting airfoil shapes were categorized by taking documented airfoil shapes, particularly from the NACA 4-digit series, and replacing their tapered leading edge with a circular one centered an input percentage along their original chord length (Fig. 2.1.1). Circular leading edges satisfied the symmetrical airfoil requirement of inverting airfoils, no matter how much the leading edge was rotated for inversion. NACA 4-digit airfoils ensured that this circular leading edge would be centered along their mean camber line and tangent to their upper and lower surfaces, due to the perpendicular bisector construction of the airfoils. The modified airfoils were then rescaled and reoriented to define a new chord line from their trailing edge to the point along the circular leading edge furthest from it. As such, a NACA 4412 5% airfoil would refer to a NACA 4412 airfoil that has been modified with a circular leading edge centered at 0.05 along its original chord of length 1, then rescaled so that this new airfoil has a chord length of 1. MATLAB code emulating these processes can be found in Appendix A.1, A.2, and A.3.

To find an inverting airfoil that both produced high lift at appropriate angles of attack (keeping the airflow attached to the airfoil to delay stall from occurring), experimental tests were conducted. In the Swarthmore College Wind Tunnel, eight inverting and five control NACA airfoils were flown to measure coefficients of lift and drag. As a result, the NACA 6412 10% airfoil was chosen for current inverting airfoil research as it produced the greatest lift at moderate angles of attack and had a low lift coefficient near a zero angle of attack (Fig 2.1.2). The MATLAB code that conducted these tests is shown in Appendix A.4.

This research was conducted from June-August 2021 under the direction of Prof. Carr Everbach in the Department of Engineering at Swarthmore College.

2.2 Non-Convexity of Bridle Support Optimization

Early design iterations of inverting airfoils assumed that the bowed-out surface of the airfoils, top in positive inversion and bottom in negative inversion, would be constrained using internal bridle strings. To solve for the best placement of these strings to induce a desired outer airfoil shape when inflated, the physical system was modeled as an optimization problem. Pressurization, like many phenomena, tends to minimize the energy density (pressure, in this case) at any given point in a bounded area by equalizing the energy. With pressure, energy minimization occurs when area is maximized. With this area maximization as an objective function to predict an inflated airfoil shape, it was discovered the problem was a quadratically constrained quadratic program (QCQP) and did not have a positive semi-definite quadratic coefficient matrix, being an NP-hard problem. As a result, the eigenfunctions of this matrix were not all positive (except for the trivial case when the coefficient matrix was of size 1x1) and the problem would always be non-convex.

In physical terms, this implicated that the pressurized airfoil could get stuck at local area maxima when inflating or given external stimuli. This would defeat the purpose of inflating to a known external airfoil shape. Instead, fabric sheets perpendicular to the wingspan of the
inverting wing were chosen to tightly constrain the outer wing profile. In turn, structural spars running this wingspan length were chosen to help support these fabric sheets.

This research was conducted from March-April 2022 with guidance from Prof. Vidya Ganapati in the Department of Engineering at Swarthmore College.

2.3 Construction of an Open Wind Tunnel

In preparation for the testing of an inverting wing for roll control, as well as other AWE kites, original equipment was fabricated in the Swarthmore College Department of Engineering (Fig. 2.3). Named the Open Wind Tunnel (OWT), a testing apparatus with an exceptionally large cross-sectional airflow was constructed from commercial fans, plywood sheets, and Unistrut steel. The existing Swarthmore College Wind Tunnel (18 in. by 24 in. cross-sectional test area) was ideal for testing small wings without internal mechanisms, but large kite experiments required cross-sectional airflows upwards of 36 in. by 120 in. The OWT was designed and constructed with these dimensions in mind, and the hexagonal stack of tapered airflow modules was fabricated atop a rolling Unistrut steel frame. With an airstream centered 54 in. off the ground, the OWT produces constant airstreams at approximately 8 mph on average.

This design project was conducted from June-August 2022 under the direction of Prof. Carr Everbach and with assistance from Jacob Sherman ’26 in the Department of Engineering at Swarthmore College.

3. Engineering Design and Methodology

To design an inverting wing that could independently morph its fabric surface near the wing root and wingtip, thus manipulate lift forces and roll moments, it was constructed entirely unlike conventional wings. The same standard aerodynamic constraints still applied to this craft: it had to withstand significant bending caused by lift or drag forces, it would attempt to reduce drag to efficiently convert most of an airstream’s energy into lift, and it would need to achieve its intended structural functions while being as lightweight as possible. To satisfy the additional physical constraints imposed by inversion, the wing would require its leading edge to rotate independently for two adjacent spans from wing root to wingtip, it had to be entirely surrounded by fabric forming an airtight seal, and it would need to position heavier elements—such as inversion motors—near the wing root as to lower its rotational moment of inertia around the roll axis, facilitating more sensitive and easily measurable wingtip inversion. For the sake of this design project and given limited space in the OWT, only one of the two wings conventional on an AWE kite was constructed to examine lift and roll properties.

To satisfy all these design requirements, the inverting wing was constructed with a unique mechanism: doubling the lateral structural beams, or spars, as drive shafts for inversion rotation. Moreover, to rotate the wingtip spars attached to the fabric leading edge with motors placed at the wing root, the trailing edge structural spar was employed as a secondary drive shaft rotationally coupled to the wingtip leading edge via pulleys. Two additional spars ran the length of the wing above and below the wing’s chord line to increase bending resistance induced by lifting loads.

Attached to these structural spars, two different varieties of internal airfoil ribs maintained the fabric wing’s exterior shape. Dictated by the optimization results presented in Section 2.2, internal fabric sheets, cut into airfoil shapes, acted in tension to maintain the wing’s NACA 6412 10% airfoil shape during full positive or negative inversion. However, to mitigate the shearing stress in the fabric at points of differential inversion—if the wingtip was in negative inversion while the wing root was positive to create a rolling moment—rigid symmetrical airfoils were placed. Created from aluminum, these airfoil frames had the same perimeter and chord length of NACA 6412 10% airfoils but were symmetrical to act as an average airfoil between positive and negative inversion shapes. Connecting these rigid airfoils to the structural spars and allowing free rotation of the leading and trailing edge spars were 3D printed supports. The supports were fixed in place with a plastic welding agent and adhered to the airfoils with heavy duty mounting tape.
Finally, the entire wing was fixed on a stationary mount in front of the OWT that allowed free rotation around the roll axis and could be manually set at different pitch angles. Attached to the mount at the wing root and counterbalanced with weights acting as the opposite wing, this setup allowed inversion forces and moments to be measured at a variety of flight angles in the OWT’s steady airstream.

This design solution addressed all the structural, functional, weight, and cost requirements imposed by aerodynamic, inversion, and material constraints. As AWE is still largely an unregulated commercial field in the wind energy market, and as inversion is an entirely original research focus in aerodynamics, no professional standards or codes add additional constraints to this design. To evaluate each inverting wing component against its guiding requirements, empirical measurement and simple observation was used. Structural elements were evaluated on their ability to withstand aerodynamic and gravity loads without deforming or twisting significantly over time; functional components on the ease of their movement along one degree of freedom and their restriction in all others; weight on whether nonessential or unnecessarily heavy items were used within the wing, especially towards the wingtip; cost on this project’s ability to be constructed under the $400 budget by reusing or repurposing other material available in the Swarthmore Engineering Machine Shop.

3.1 Inverting Wing Geometry

The outer geometry of the inverting wing was determined by classical aerodynamic principals, inversion constraints, and material availabilities. The planform shape of the wing was originally proposed to taper linearly from wing root to wingtip but was later changed to exclude taper, ensuring a constant cross-section size along the wing for ease in assembly. With a rectangular wing planform, the aspect ratio of wingspan to chord length was determined to be in the range of 6 to 10 [5]. Greater aspect ratios reduce the induced drag on a wing, caused by wingtip vortices, but the benefits of ratios above 10 are negligible.

The chord length of the wing, from leading to trailing edge, was chosen to be 24 inches so that the corresponding NACA 6412 10% airfoil height at roughly 3.5 inches would be large enough for a human hand to fit inside during assembly. As only half of a full kite was constructed for this project, the length of this wing could range from 6 to 10 feet to meet aspect ratio requirements. These dimensions would also fit within the width of the OWT testing section.

In order for the wingtip to produce enough force to create a moment about roll axis centered at the wing root, it had to overcome the moment created produced by the wing root. Assuming a uniformly distributed aerodynamic load w induced by an airstream parallel to the chord line of the wing, the length x from the wing root to the point between the difference in inversion can be described by the equation:

$$M = w \frac{x^2}{2} - w(\frac{1}{2} + \frac{1}{2})(1 - x)$$

It is assumed in this configuration that the wingtip inversion attempts to provide a greater moment on the wing than the main wing itself to roll the entire setup, producing an overall negative moment. It then follows that $M < 0$ for $0 < x < \sqrt{2}/2$, but the wingtip inversion adds more sensitivity in roll closer to this upper bound. Thus, the point $x$ between positive and negative inversion can be taken at or below $\frac{1}{2}$ the wingspan.

Carbon fiber spars are only available at low cost at predetermined lengths, and the longest single spar length produced by the manufacturer goodwinds.com is 5 ft. For the full wing to comfortably fit within the width of the OWT and so that multiple carbon fiber sections would be joined lengthwise near the wingtip and wing root connection, the wing was chosen to measure 7.5 ft. in length (Fig. 3.1). This ensured the aspect ratio of the full wingspan, double the length of this right wing, would be 7.5 within the 6 to 10 bounds. This length would also ensure the full 5 ft. of a single carbon fiber rod could be used in the wing root section, connected to a 2.5 ft. rod to support the wingtip section.

3.2 Carbon Fiber Spars

Four carbon fiber spars, 7.5 ft. in conjoined length, were positioned inside the inverting wing as structural beams and transmission shafts to drive inversion. All spars were created by taking six 5 ft. lengths of 0.394 in. by 0.288 in. carbon fiber tubes, cutting two of them in half to create 2.5 ft. segments, and permanently bonding three sets of the 5 ft. and 2.5 ft. tubes. For a solid connection between these lengths to reinforce the 7.5 ft. wing, 3 in. segments of 0.500 in. by 0.402 in. carbon fiber tubes were telescoped over the connection point of the 0.394 in. spars and adhered to both
The moment of inertia to resist bending along the vertical lift axis, the spars were offset as far from the chord line as the inverted fabric exterior would allow, at 0.75 in. each. Compared to a single spar placed along the chord line of the wing, these two offset spars decreased theoretical wingtip deflection from 23.6 in. to 0.0375 in. under a 25 N/m distributed load (Fig. 3.2.1(b)).

The final two 7.5 ft. spars were placed along the leading and trailing edges of the wing to both turn the inversion at the wingtip and wing root as well as provide bending resistance to drag forces. As previously discussed, the leading edge spar was not bonded together, unlike all other spars, but allowed to rotate independently along the wingtip and wing root segments via needle nosed-nosed bearings. The trailing edge spar segments were bonded together but was still allowed to rotate freely in its 3D printed supports with additional bearings (Fig. 3.2.2). By later attaching pulleys and a V-belt to the trailing edge spar and the wingtip portion of the leading edge spar, the trailing edge spar could effectively control wingtip inversion. In future research, this will allow inversion motors to be positioned in the center of an inverting kite, centralizing its mass, lowering its rotational moment of inertia along the roll axis, and subsequently letting wingtip inversion more sensitively roll the kite in flight.

However, without torsional resistance on these leading and trailing edge spars, only the center structural spars would be able to resist twist along the axis of the wing. Since twist resistance would require another 7.5 ft. spar fixed to the wing frame, the cost savings without this spar outweighed the perceived benefits of torsional bracing.

3.3 Rigid Airfoil Ribs

Four sets of thin aluminum frames, or ribs, were positioned around the carbon fiber spars. Two were set at the start of the wing root, one at the end of the wingtip, and vertically a quarter of the chord length inside the wing, where lift is most commonly centered [5]. To increase the moment of inertia to resist bending along the vertical lift axis, the spars were offset as far from the chord line as the inverted fabric exterior would allow, at 0.75 in. each. Compared to a single spar placed along the chord line of the wing, these two offset spars decreased theoretical wingtip deflection from 23.6 in. to 0.0375 in. under a 25 N/m distributed load (Fig. 3.2.1(b)).

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one two-thirds along the wing at the intersection of the wing root and tip inversion sections. The wingtip rib acted as a rigid support to cover with fabric and ensure a stationary airtight seal; the middle acted as an average airfoil between positive and negative inversions to reduce shearing stress on the surface fabric; the wing root ribs acted both as a rigid surface for a wing-end seal as well as two attachment points to transfer the internal wing moment onto the freely rotating mount axis.

Each rib was created by bending a 0.125 in. by 1 in. by 48 in. stock aluminum strip around a wooden block—cut via band saw into a symmetrical airfoil—to achieve a desired shape (Fig. 3.3). Aluminum, like all materials, springs back towards its pretensioned shape even after permanent deformation. To accommodate this, the ribs had to be bent by hand beyond the airfoil shape until the unstressed frame matched the wooden profile.

The exact symmetrical airfoil shape of these ribs was arbitrarily chosen within guiding constraints, both because an ideal symmetrical airfoil shape would be numerically difficult to model and because the aluminum bending processes were too imprecise to match such an ideal airfoil. The four chosen constraints for this airfoil shape were (i) the chord length of the symmetrical airfoil had to match the chord length of the inverting airfoil at 24 in., (ii) the perimeter of the symmetrical airfoil had to match the perimeter of the inverting airfoil at 49.81 in., (iii) the point along the top or bottom symmetrical airfoil surface that was the farthest distance from the chord line had to occur a quarter along the length of the chord, like the inverting airfoil, and (iv) the leading edge of the symmetrical airfoil had to have a circular arc of radius 1.17 in. to allow inversion. A numerically ideal symmetrical airfoil for inversion would require the minimization of the difference in distances between points along the symmetrical airfoil and an inverting airfoil in positive and negative inversion.

This would reduce the strain and subsequently stress in the fabric between these two difference airfoils when morphed for inversion.

Stock aluminum strips were only available in lengths of 48 in., so the top and bottom surfaces of the ribs were left unconnected at the trailing edge. This later allowed interior maintenance of the wing after the exterior fabric had been applied and before the trailing edge had been sealed.

3.4 3D Printed Connectors

To connect the carbon fiber spars to the aluminum ribs while allowing free rotation along the leading and trailing edge spars, three sets of 3D printed supports were placed on each of the four ribs (Fig 3.4(a)). The 3D printed supports on the leading and trailing edge were identical: they consisted of hinged legs pivoting around a hollow shaft, within which two 12mm needle-nosed bearings were bonded to with J-B plastic welding epoxy (Fig. 3.4(b)). The two bearings allowed the carbon spars to rotate while preventing shimmy. To restrict the spars’ lateral movement within the bearings, 3D printed stoppers were bonded to the spars on either side of the bearings at each rib. Finally, the 3D printed supports were hinged to ensure a parallel connection to the aluminum ribs even when the exact slope of the rib was unknown. These PLA-aluminum connections were stuck together with 1 in. by 1 in. squares of double-sided mounting tape.

The 3D printed pieces supporting the central structural spars did not need to facilitate free rotation, so they were bonded with J-B plastic welding epoxy directly to the carbon fiber. However, they did need to precisely space the structural spars 1.5 in. from one another, centered exactly along the wing’s chord line. The resulting 3D printed part had two hinged joints around the spars to allow for varying aluminum rib slopes, shown in Fig. 3.4(c).

3.5 Fabric Airfoil Sheets

Eight rows of ripstop nylon sheets, cut into NACA 6412 10% airfoil shapes with protruding tabs along the cambered upper surface, transferred lifting force from the wing surface to the lateral spars between aluminum ribs. Five fabric supports were designated for the wing root inversion section; the other three supported the wingtip portion (Fig 3.5(a)). Each row of sheets was made by sliding two fabric airfoils, faced up and down, onto the carbon fiber spars with 0.5 in. grommets. The airfoils facing separate directions allowed one fabric sheet to be taut for positive inversion and the other for negative. When the inverting wing changed inversion, the unused fabric sheet folded inside the wing instead of obstructing the wing’s morphing.

Tabs along the outside portion on the fabric airfoils were adhered to the exterior fabric surface with thin strips
 allowable locations for tabs were numerically computed by finding all locations along the upper surface of the inverting airfoil where the distance to the upper carbon fiber spar was shorter during positive inversion than in negative inversion. In other terms, all points that moved towards the upper support and not away during positive inversion were desired to prevent strain in the supporting fabric. By a symmetry argument, these locations would also work for the lower inverting wing airfoil tabs. The inequality that solves for this portion of the NACA 6412 10% upper perimeter is shown in red in Fig 3.2.1, MATLAB code in Appendix A.5.

Once all points on the NACA 6412 10% airfoil that satisfied this inequality had been identified in MATLAB, the solution curve and airfoil was converted into a .DAT file, imported into the CAD modeling program Fusion 360, and made into a sketch. Tabs protruding 1 in. and measuring approximately 3 in. in length were offset from the solution curve, 0.5 in. holes were placed where the carbon fiber spars would slide through, and finally the entire sketch was exported as a .DXF file. Next, the file was converted online into an SVG format, imported into the digital art software Adobe Illustrator with a scaling factor of 96 (to convert pixels to inches) and line widths of 0.01, and cut on an 28in. by 40 in. Epsilon Laser Cutter with settings of 60% speed, 50% power, and 10% frequency (Fig 3.5(b)).

3.6 Wheels and Pulleys for Inversion

Seven 3D printed wheels with outer diameters of 2.34 in. and inner diameters of 0.402 in. were bonded to the leading edge spar with J-B plastic welding epoxy. Each wheel featured six spokes connecting the two circular profiles, all with a 3D printed thickness of 0.080 in. Four wheels were placed uniformly along the wing root portion of the inverting wing; the other three were spaced along the wingtip segment. When the outer fabric was subsequently wrapped around the entire wing, strips of 2 in. double sided mounting tape connected the front of every wheel to the fabric. This ensured that the rotating leading edge could manipulate the fabric surface through the leading edge wheels. To reduce the shearing stress in the fabric remained at a minimum when pulled between the rigid aluminum ribs and the leading edge wheels, the wheels closest to the ribs were placed 6 in. away from them along the spars. This would theoretically produce a 0.0138 in./in. strain in the fabric, which would be well within the elastic stress region of ripstop nylon [6].

Originally, two 3L belts were intended to be wrapped around 0.375 in. 3L zinc pulleys at the start and end of the wingtip inversion section along the leading and trailing edge spars. To fit on the 0.394 in. diameter carbon fiber tubes, the pulleys shafts were widened to a diameter of 0.402 in. using a lathe. With the relative malleability of zinc compared to hardened steel drill bits, this also ensured the set screw hole attaching the pulley to the spar would remain

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**Fig 3.4** from the top down (a) three sets of 3D printed supports connected the aluminum ribs to the carbon fiber spars, (b) each support along the leading and trailing edge accommodated two needle-nosed bearings to let the spars rotate freely along their axis, (c) the supports for the central spars were rigidly held and bonded with a plastic welding agent directly to the spars to prevent slip.
Fig. 3.5  from the top down (a) orange ripstop nylon fabric sheets are placed within the inverting wing to constrict its exterior surface into a NACA 6412 10% shape. (b) fabric sheets were designed in Fusion 360 and cut on an Epsilon 28 in. by 40 in. laser cutter.

accessible. These pulleys would allow the trailing edge to be rotationally coupled to the wingtip portion of the leading edge, driving inversion. However, the 44 in. circumference belts ordered were discovered to be approximately 0.5 in. shorter than needed, with little flexibility during rotation. The pulleys were instead moved to the exterior of the wingtip so that future experimentation with wingtip inversion could be accessed outside the closed-in wing. It is suggested that these future iterations use timing pulleys with cogged belts to wrap around the small turn radius.

3.7 Ripstop Nylon Wing Surface

The fabric wing surface, made from a single sheet of ripstop nylon measuring 90 in. by 52 in., was pressed onto the double-sided mounting tape already connected to the aluminum ribs and fabric airfoil sheets. After all tape had been firmly adhered to the bottom of the wing, the wing was rolled over its leading edge and connected along its top side. Note that the fabric airfoil sheets were connected with splayed tabs alternating in perpendicular direction from the airfoil vertical placement, as discussed in Section 3.5. The fabric on the trailing edge was then sealed to itself using a continuous 90 in. strip of double-sided mounting tape, and the fabric at the wingtip and wing root aluminum frames were temporarily sealed together with duct tape. This allowed future maintenance to occur within the wing while forming a good airtight seal during testing.

3.8 Internal Pressurization

A small commercial 130W air pump with a maximum airflow of 440L/min. was fed into the fabric along the root side of the wing. Though pressurized AWE kites do not typically use active air pumps to inflate themselves, airtight seals were not expected with this tested inverting wing. The air pump instead blew a constant airstream into the wing, equilibrating at a constant pressure when the airflow into the wing matched the amount leaking out.

3.9 Wing Testing Mount

A wooden testing mount to hold the wing was constructed with available 2 in. by 4 in. lumber, repurposed PVC pipes, locking swivel wheels on hand, and purchased ball bearings. The mount had to satisfy a number of design requirements to properly test the inverting wing: it had to (i) allow the wing to freely rotate around an axis parallel to the wing’s chord line, (ii) accommodate manual adjustment of the pitch angle of the wing, effectively controlling angle of attack for all roll tests, (iii) effectively distribute the varying vertical forces from lift and self-weight onto four swivel wheels, and (iv) prevent tipping from excessive lateral forces imposed if the wing or a user were to crash into it.

Fig 3.9  The top of the wooden wing mount is pictured with the inverting wing on the right and counterbalance on the left.
The final mount design used a 0.750 in. by 0.540 in. by 24 in. carbon fiber tube connected to R12 0.75 in. ball bearings as the roll axis for the wing (Fig. 3.9). Each bearing was set into counterbored halves of 2 in. by 4 in. wooden half-lap joints set at right angles so that the bearing was fully enclosed to transfer forces without impairing the rotational freedom of the piece. The two right angle joints were placed at the top of a 2 in. by 4 in. wooden frame to form a slanted roof shape; the frame was finally joined to a wide wooden base with an aluminum shaft perpendicular to roll axis. This allowed the frame, and thus wing, to rotate freely along the pitch axis. To constrain rotation in this direct except for when a user wanted to change the pitch angle, two sets of PVC pistons were hinged between the frame and the rolling base. With two lubricated 3/16 in. fractional 326 O-rings in each piston between tubes of 1.25 in. and 0.75 in. PVC pipe, held in place with standard 0.75 in. PVC connectors and end caps, the airtight pistons only moved when applied with a force exceeding roughly 50 lb. As a result, the wing could not produce enough force to change this pitch angle; it was effectively constrained.

3.10 Wing Attachment to Mount and Counterbalance

Two sets of machined aluminum soft jaws clamping around the two central carbon fiber spars were bolted to two long 0.25 in. by 2 in. by 36 in. aluminum strips (Fig 3.10). With their two attachment points to the wing via the structural spars, the strips effectively transferred the force and moment created by the wing onto the wing mount axis through milled 0.75 in. diameter holes. Both strips were oriented so that their 2 in. dimensions were vertical, increasing the attachment’s moment of inertia and decreasing deflection. After the two strips were connected to the wing mount, a wooden counterbalance with a 2 in. by 4 in. cutout was slid onto the strips. This counterbalance included a long slot perpendicular to the aluminum strips’ axes to allow for steel Unistrut pieces to be stacked. As a result, the counterbalance’s weight and position relative to the roll axis could be precisely controlled.

In future testing, once the wing has been inverted at a certain pitch angle in the OWT’s airflow, the counterbalance will be stacked and positioned to equalize the wing’s moment. By subtracting the moment the wing produces without an airstream at the same pitch angle, the inversion moment and thus force can be solved for. Given the sensitivity of the roll axis, this force measurement should be precise.

4. Results and Discussion

This engineering capstone project was evaluated on its ability to fulfill the imposed structural, functional, weight, and cost requirements imposed on the construction of an inverting wing testing setup. Previously, the criteria for success of this project hinged on the demonstration of four key aspects: (i) the robustness of the wing mount structure and counterweight as an apparatus to test the roll control of any wing in the OWT at different pitch angles, (ii) the feasibility of the inverting wing with roll control made in this project for its application on AWE kites, (iii) the precision of autonomous inversion motors and algorithms to keep the inverting wing level in flight using wingtip roll control, and (iv) the force and moment measurements of the inverting wing in different flight configurations at various pitch angles. As the inversion motors and algorithms were left to future implementation, the third and fourth goal of this project were combined into the ease at which force and moment measurements could be obtained once inversion motors are installed.

The wing mount and counterbalance setup performed better than initially designed for. The roll axis on which the wing and counterbalance were mounted spun with
almost negligible friction or deflection in the axis. When small nudes were applied to the wingtip with forces at a fraction of a Newton, the entire wing spun at rotational speeds less than a degree per second for extended periods of time. If noticeable rotational friction was apparent, the wing would have either stopped after a short period of decelerating rotation or not have moved at all. This unconstrained degree of freedom is likely due to the combined effects of the 0.75 in. ball bearings being well aligned at the ends of the roll axis and the machined 0.75 in. slots that the axis fit through to hold the wing, providing two places to reduce rotational friction.

The manual pitch of the wing mount also performed better than intended such that the initial design was simplified during construction. Originally, the telescoping, airtight PVC pistons that guide the pitch angle were to be complemented by a locking mechanism like in extendable ladders; it was assumed the pistons could not support the weight of the wing without sliding. However, the static friction of the pistons was so much greater than their kinetic friction that only forces exceeding roughly 50 lb. could change the pitch angle, necessitating human intervention. As a result, only pistons were used to support and change the wing mount pitch.

The entire wing mount was constructed at a very low cost: it only required the purchase of two ball bearings and one carbon fiber tube. All the wooden 2 in. by 4 in. lumber, the PVC piston components, the swivel wheels, and the pitch aluminum shaft were repurposed from other projects in the Swarthmore Engineering Machine Shop. The frame was also structurally sound beyond expected loads: it exhibited no tip under lateral loads from the wing and was even able to serve as a platform to lower thermocouple wire in the Swarthmore Geothermal Well Field across the campus. As such, its low weight allowed it to be transported across campus with ease.

Before the outer fabric was applied to the inverting wing, the leading and trailing edge spars were able to rotate with very little rotational friction. After each of the two spars had passed through eight needle-nosed bearings and sixteen fabric sheets, each requiring 3D printed stoppers and plastic welding agents to restrict lateral movement, nearly free rotation was an unexpected yet desirable outcome. Had excess plastic welding epoxy touched both the 7.5 ft. spar and any one of these twenty-four components, the spar would not have been able to turn.

The central spars serving as structural bracing also performed admirably, reducing deflection of the wingtip to less than 2 in. over the entire 90 in. span. However, as both leading and trailing edge spars could not resist torsional movement—they were designed to rotate freely—the entire wing twisted noticeably over its span to nearly a 20° angle difference between wingtip and wing root. For the sake of cost reduction in this project, an additional spar near the trailing edge to reduce torsion was deemed unnecessary. The carbon fiber spars constituted more than half the cost of this project, but in future inverting designs a torsional bracing spar is recommended.

The weight of the entire wing was acceptable, at only a few pounds without the metal soft jaws to connect it to the wing mount, but the bulk of weight was taken up by the aluminum ribs and double-sided mounting tape. Neither of these materials are essential for inverting wings and both should be replaced in future design iterations. The 0.25 in. thick aluminum for the ribs was chosen for its versatility in bending and its availability in the machine shop to reduce costs. Future ribs should either be 3D printed, cast in thin molds, or made from 0.0625 in. aluminum strips. The mounting tape should be replaced with plastic welding epoxy between solid elements and stitching for fabric connections. Both replacements would significantly increase the time spent in fabricating the wing and possibly increase costs, but they would reduce the wing to a fraction of its current weight.

The functionality of the inverting wing was not immediately discernable after the exterior fabric had been wrapped around the interior supports and leading edge wheels. The internal pressurization certainly provided sufficient inflation; the entire wing noticeably plumped up with a smooth surface after turning the air pump on and was firm to the touch. Turning the leading edge spars by hand, too, noticeably morphed the exterior wing fabric. As expected of inversion, spinning the wheel in one direction pulled in fabric along the top or bottom surface and paid it out to the other side. However, without inversion motors to provide constant torque on the spars, the wing could not stay inverted without human intervention. As a difference in lift and thus roll direction would be required to validate if this inverting wing exhibited inversion at wing root or wingtip, exterior force would not be allowed. So, without inversion motors to continuously torque the leading edge, inversion lift could not be determined. The next step for this research on using inversion as a roll control mechanism would be to implement torque motors and measure the change in lift when the leading edge spar is turned. From there, a full inverting AWE kite should be constructed and flown for test data when tethered and subjected to unsteady airstreams.

5. Summary and Conclusion

This Swarthmore Engineering senior capstone project designed, constructed, and evaluated the use of inverting wing mechanisms for roll control on AWE kites. In doing so, it fulfilled the design requirements for functionality in inversion, structural integrity under expected loads, and reduction in overall weight and cost. These constraints were explicitly analyzed for the success of the wing mount setup, the inverting wing, and the force measurement system to later characterize the lift and roll produced by the wing. As a summary of these results, this project:
1. Efficiently constructed a wing mount setup with a low friction roll axis at low cost.
2. Developed a manual pitch system on the wing mount for structure and function.
3. Integrated carbon fiber spars into the wing as drive shafts and deflection bracing.
4. Reduced the complexity of components in the wing to lower weight and cost.
5. Designed a precise counterbalance method to measure the wing’s lift and moment.
6. Successfully inflated and flew the wing in the OWT with unconstrained roll.
7. Requires torque motors driving the inversion spars to measure lift during inversion.

This project came to an overall cost of $410.56, just above the desired $400 benchmark, where 56% of this expense came from carbon fiber tubes. Future developments of this inverting kite project are recommended to first attach timing pulleys and cogged belts from the leading edge spar at the wingtip to the trailing edge spar, measure the holding torque required to invert the wing through the leading and trailing edge spars at the wing root aluminum rib, and choose lightweight motors with similar torque curves. From there, measure all combinations of positive and negative inversion at wingtip and wing root at a variety of pitch angles. To create a full inverting kite, segment two copies of the inverting wing created in this project to a central chassis with four inversion motors, a microcontroller, and an energy storage system. Replace the thick aluminum ribs and separate spar supports with continuous 3D printed parts, adhering all sections with plastic welding glue. Finally, add wingtips to kite, attach a pitch controlling tether, and test it in outdoor flight.

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References

Appendix A: MATLAB CODE

Appendix A.1: NACA.m

function [xU,yU,xL,yL,xc,yc] = NACA(four_digit_number,spline_points,plot_onoff)
% written by Josh Vandervelde for Summer Research on 6/15/2021
% generates and plots a spline for the input NACA four-digit airfoil
% returns x,y coordinates for both upper and lower airfoil surfaces
% equations are created by NACA, found on wikipedia.org/wiki/NACA_airfoil
% for more information on NACA airfoil characteristics: airfoiltools.com
% NACA digits describe:
% 1st digit: maximum camber (from chord) in hundredths of chord length...
% For NACA 6412, maximum camber is 6% of chord length
% 2nd digit: distance of maximum camber from airfoil leading edge in...
% tenths of chord length. For NACA 6412, maximum camber is 40% of...
% the way along the chord from the airfoil leading edge
% 3rd & 4th digit: maximum thickness of airfoil in hundredths of chord...
% length. For NACA 6412, maximum thickness is 12% of chord length
% spline_points: number of returned coordinates on either airfoil surface

% Separating NACA Number Into Component Parts
m = floor(four_digit_number/1000)/100; % 1st digit
p = floor((four_digit_number-m*10^5)/100)/10; % 2nd digit
t = (four_digit_number-m*10^5-p*10^3)/100; % 3rd-4th digits
xc = linspace(0,1,spline_points); % x values between 0 and 1

if m >= 0.1 % removing this flag allows for higher maximum camber (if wanted)
    error('Error: NACA airfoil code is not a four-digit number');
end

% Separating x Values For Piecewise y_c Function
x1 = xc(xc<p);
x2 = xc(xc>=p);
yc1 = m*(2*p*x1 - x1.^2)/p^2;
yc2 = m*((1-2*p) + 2*p*x2 - x2.^2)/(1-p)^2;
dyc1_dx = 2*m*(p-x1)/p^2;
dyc2_dx = 2*m*(p-x2)/(1-p)^2;
theta1 = atan(dyc1_dx);
theta2 = atan(dyc2_dx);

% Rejoining Later Used Piecewise Values
yc = [yc1,yc2];
theta = [theta1,theta2];

% Half The Thickness of The Airfoil, y_t
yt = 5*t*(0.2969*xc.^0.5 - 0.1260*xc - 0.3516*xc.^2 + 0.2843*xc.^3 - 0.1036*xc.^4);

% Lower and Upper Surface Coordinates
xU = xc - yt.*sin(theta);
xL = xc + yt.*sin(theta);
yU = yc + yt.*cos(theta);
yL = yc - yt.*cos(theta);

% Plotting Airfoil Shape
if plot_onoff == 1
    airfoil_name = strcat('NACA','{','}','Airfoil');
    figure('Name',airfoil_name,'NumberTitle','off');
hold on
p1 = plot(xc,yc,'Color',[200,200,200]/255,'LineWidth',2); % camber
p2 = plot(xc,0*xc,'--','Color',[200,200,200]/255,'LineWidth',2); % chord
p3 = plot(xU,yU,'k','LineWidth',2); % upper surface
p4 = plot(xL,yL,'k','LineWidth',2); % lower surface
hold off

title(airfoil_name);
xlabel('Chord Length');
ylabel('Thickness');
legend(p3,p1,p2,'Airfoil Surface','Camber Line','Chord Line');
grid on;
grid minor;
axis equal;
end
function [xU,yU,xL,yL,circle,turn_angle,excess_fabric] = NACAleadingcircle(NACA_airfoil,circle_x,saveonoff)
% NACAleadingcircle.m
% written by Josh Vandervelde for Summer Research on 6/22/2021
% plots a NACA airfoil with a circular leading edge and flat bottom (for inverting airfoils)
% inputs:
%   NACA_airfoil: four-digit code for NACA airfoil
%   circle_x: x coordinate of leading edge circle center as a fraction of chord length
%   saveonoff: 1 to save data to .dat file, anything else to not save
% outputs:
%   xU,yU,xL,yL: x and y coordinate arrays for upper and lower surfaces of airfoil with circular leading edge
%   circle: 1x3 array of circle x and y center coordinates and its radius
%   turn_angle: angle (degrees) front wheel needs to turn to fully invert airfoil
%   excess_fabric: length of material that should be pinned down to leading circle starting at the upper surface

fabric connection
%Generating NACA airfoil
[xU,yU,xL,yL,sc,yc] = NACA(NACA_airfoil,1000,0); %change to input variable spline points if need be
shifted_xU = [0,xU(1:end-1)]; %subtract from xU to find array of distances between xU values
shifted_yU = [0,yU(1:end-1)]; %subtract from yU to find array of distances between yU values
all_u = ((xU-shifted_xU).^2 + (yU-shifted_yU).^2).^0.5; %approximated length of upper surface

%Indices of Circle intersection of camber, upper surface, lower surface
ind = find(xc>circle_x); %indices of all camber x coordinates greater than circle center
i = iind(i); %index of closest camber point to circle center
centerXY = [circle_x,yc(i)]; %coordinates of circle center along camber line
ind = (xU-centerXY(1)).^2 + (yU-centerXY(2)).^2 < turn_angle; %distances from circle center to all points on upper surface
[r1,j] = min(jind); %minimum distance from circle center to upper surface and corresponding index
upperXY = [xU(j),yU(j)]; %coordinates of where the circle will be tangent to the upper surface
kind = (xL-centerXY(1)).^2 + (yL-centerXY(2)).^2 < turn_angle; %distances from circle center to all points on lower surface
[r2,k] = min(kind); %minimum distance from circle center to lower surface and corresponding index
lowerXY = [xL(k),yL(k)]; %coordinates of where the circle will be tangent to the lower surface

%Circle characteristics
r = mean([r1,r2]); %radius of front circle based on camber to surface distance
q = (upperXY(2)-lowerXY(2))/2 + (upperXY(2)-lowerXY(2)^2)/2; %arclength of exposed leading edge of circle
u = sum(all_u(j:end)); %length of upper surface from circle to trailing edge
l = ((1-lowerXY(1))^2 + lowerXY(2)^2)^0.5; %length of lower surface from circle to trailing edge (assuming a straight line)
circle = [centerXY,r]; %coordinates and radius of circle

%Checking to see if this circle size will work on airfoil
excess_fabric = s-l-u; %this can be commented out if not needing inverting airfoils
if excess_fabric < 0
    error('Error: circle position is too small for airfoil inversion to have enough fabric.'
        '</excess_fabric = %4.4f, excess_fabric');
end

%Finding the angle to turn front wheel for full inversion
turn_angle = round(360*(s-excess_fabric)/(2*pi*r),2); %arc length turned to radians turned to degrees

%Generating Top and Bottom Semicircles
turn_angle = [0:0.001:2*pi]; %front half of circle
x_circle = r*sin(angle) + centerXY(1); %x points for circle
y_circle = r*cos(angle) + centerXY(2); %y points for circle

%leftover Airfoil Surfaces
xU_old = xU(1:j);
yU_old = yU(1:j);
xL_old = xL;
yL_old = yL;

%Designating Compressed Airfoil Surfaces
xU_comp = [flip(xUcircle(1,1:1000:end)),xUcircle(1:1000:end)]; %include a fraction of original points to reduce fusion runtime
yU_comp = [flip(xUcircle(2,1:1000:end)),yUcircle(1:1000:end)]; %include a fraction of original points to reduce fusion runtime
xL_comp = [flip(xLcircle(1,1:1000:end)),xLcircle(1:1000:end)]; %more lower spline points
yL_comp = [flip(xLcircle(2,1:1000:end)),yLcircle(2,1:1000:end)]; %more lower spline points

%Saving to Dat File
if saveonoff == 1
    xy = [[xU_comp, flip(xL_comp)],[yU_comp, flip(yL_comp)]]; % all compressed x and y coordinates around airfoil
    filename = strcat('NACA_', string(NACA_airfoil), '_Circle_', string(round(circle_x*100)), '_Percent.dat');
    save(filename, 'xy', '-ascii'); % data saved to relevant name i.e. 'NACA_6412_Circle_10_Percent.dat'
end

% Plotting Airfoil
airfoil_name = strcat('NACA', {''}, string(NACA_airfoil), {' '}, 'Airfoil Circular Leading Edge At', ...% Of Chord Length
{' '}, string(circle_x*100), '{' '), 'k', '{' '), 'Color', [200, 200, 200]/255, '{' '}, 'LineWidth', 2); % leading edge full circle
p5 = plot(xU, yU, 'k', '{' '}, 'LineWidth', 2); % new upper surface
p6 = plot(xL, yL, 'k', '{' '}, 'LineWidth', 2); % new lower surface
% p7 = plot(xU_comp, yU_comp, {'.'});
% p8 = plot(xL_comp, yL_comp, {'.'});
hold off
grid on;
grid minor;
axis equal;
end

Appendix A.3: normalize_airfoil.m

% takes an arbitrary airfoil shape, rotates it, and scales it to have its
% chord along the x-axis from 0 to 1. Outputs an 2xn array of airfoil coords
% created by Josh Vandervelde on 10/2/2022

function [x,y] = normalize_airfoil(xU,yU,xL,yL)
% Rotate, Translate, and Scale Airfoil to Have Chord Length 1 Along X-Axis on the Origin
airfoil1 = [xU, flip(xL); yU, flip(yL)];
[~, max_ind] = max(airfoil1(1,:)); % find maximum difference
theta = -atan((airfoil1(2,max_ind) - airfoil1(2,min_ind))/(airfoil1(1,max_ind) - airfoil1(1,min_ind)))); % angle of chord relative to x-axis
airfoil2 = [cos(theta), -sin(theta); sin(theta), cos(theta)]*airfoil1; % rotate airfoil to align with x-axis
airfoil3 = [airfoil2(1,1) - airfoil2(1,min_ind); airfoil2(2,1) - airfoil2(2,min_ind)]; % move front of airfoil to origin
airfoil4 = airfoil3/chord; % scale the airfoil based on its previous chord length to have new length 1
x = airfoil4(1,:); y = airfoil4(2,:);
Appendix A.4: airfoil_energy.m

```matlab
function [a,Cl,Cd,Re,w,h] = airfoilenergy(NACA_airfoil,circle_x,chordlength,wingspan)
% finds airfoil lift/drag coefficients and plots resultant forces against the airfoil L/D ratio (applications in tethered airfoils)
% written by Josh Vandervelde on July 21, 2021 for Summer Research in Inverting Airfoils

% inputs: all single numbers
%   NACA_airfoil: four-digit code specifying the NACA airfoil model (refer to script 'NACA.m' for a full breakdown of NACA codes)
%   circle_x: x coordinate of leading edge circle center as a fraction of chord length (for modified airfoils; if unmodified, input '0')
%   chordlength: distance from airfoil leading edge to trailing tail (meters, typically 0.2 for Swarthmore Wind Tunnel)
%   wingspan: all row vectors of equal length (indices matching angles of attack)
% % outputs: all row vectors of equal length (indices matching angles of attack)
%   a: alpha, the angles of attack airfoil characteristics are measured at (degrees)
%   Cl: lift coefficients, dimensionless values indicating general lift characteristics of tested airfoil
%   Cd: drag coefficients, dimensionless values indicating general drag characteristics of tested airfoil
%   Re: Reynolds Numbers, dimensionless values indicating fluid flow characteristics of wind tunnel tests versus larger scaled models
%   w: projected widths of the airfoil as seen relative to wind moving on the x-axis (max(x)-min(x) as fractions of airfoil chord length)
%   h: projected heights of the airfoil as seen relative to wind moving on the x-axis (max(y)-min(y) as fractions of airfoil chord length)
% saved files:
%   '.fig' file: two plots are saved, one of lift coefficients versus drag coefficients and the other of standard resultant forces versus lift/drag ratios
% Set Up Airfoil and Arrays
a = linspace(0, 15, 25); % angles of attack (degrees) to test airfoils at from -15 to 15 with 2.5 steps
input('Set the airfoil’s chord parallel to wind direction, then rotate the turn plate to ''string(round(a(1),2)),'' degrees. Press ENTER to continue.');
num_angles = length(a); % number of angles to test through

%NACA airfoils: four-digit code specifying the NACA airfoil model (refer to script 'NACA.m' for a full breakdown of NACA codes)
%calibrating Force Balance
RPMS = [300,500,700]; % wind velocities to test airfoils at, corresponding to about 6, 10, and 14 m/s
rho = 1.2041; % density of dry air at 20C
nu = 1.640e-5; % kinematic viscosity of air at sea level
load gong.mat % or 'load handel.mat y' if you want to spice the sound up

% Generate Points Along Airfoil Surfaces
if circle_x == 0
    [xU,yU,xL,yL,r,~,~] = NACAleadingcircle(NACA_airfoil,circle_x,0);
else
    [xU,yU,xL,yL,r,~,~] = NACAleadingcircle(NACA_airfoil,circle_x,0);
end
chordlength = chordlength*(1-chordlength); % scale leading edge airfoils to appropriate chordlength, comment out if inputing measured chordlength (not 0.2 meters)

% Rotate, Translate, and Scale Airfoil to Have Chord Length 1 Along X-Axis on the Origin
airfoil = [xU,flip(xL);yU,flip(yL)];
[-max_ind] = max(abs(airfoil(1,1,1))); %max of airfoil, airfoil(1,1,1));
[chord,min_ind] = max((airfoil(1,1,:)-airfoil(1,min_ind)).^2); % find max/min x and y values (projected width and height)
theta = atan((airfoil(2,max_ind)-airfoil(2,min_ind))/(airfoil(1,max_ind)-airfoil(1,min_ind))); % angle of chord relative to x-axis
airfoil = [cos(theta),-sin(theta);sin(theta),cos(theta)]*airfoil; % rotate airfoil to align chord with x-axis
airfoil = airfoil/chordlength; % scale the airfoil based on its previous chord length to have new length 1

% Determine Projected Width(w) and Projected Height(h) of Rotated Airfoil
for i = 1:num_angles
    rotation_matrix = [cos(a(i)),-sin(a(i));sin(a(i)),cos(a(i))]; % all angles are neg as pos attack rotates clockwise
    rotated_airfoil = rotation_matrix*airfoil;
    w(i) = max(rotated_airfoil(1,:))-min(rotated_airfoil(1,:)); % distance between max/min x values (projected width)
    h(i) = max(rotated_airfoil(2,:))-min(rotated_airfoil(2,:)); % distance between max/min y values (projected height)
end

% Calibrating Force Balance
if RPM() < 1 % calibrateFB();
else
    RPM(1); pause(7); RPM(0);
    calibrateFB();
end
fprintf('%n');

% Running Experiment
for i = 1:num_angles
    if i>1
        soundsc(y); % play sound to notify user when to turn the plate
    end
    fprintf('Rotate the turn plate 2.5 degrees to the
    ','string(round(a(i),2)),
    % "marking. Press ENTER to continue.
    ');%for j = rpms % RPM speeds, or about 6, 10, and 14 m/s wind velocity
    % input('Rotate the turn plate 2.5 degrees to the ','',' marking. Press ENTER to continue.
    %for j = rpms % RPM speeds, or about 6, 10, and 14 m/s wind velocity
    elseif i == 1 & j == rpms
        pause(10); % initial startup of fan to 300 RPM
    end
end
```

drag = abs(drag)/10; % drag should always be positive, seems to be receiving significant negative values once in a while at 10x magnitude of expected (factor of g)
velocity = freeflow();
test_Cl(j==rpms) = (2*lift)/(w(i)*wingspan*chordlength*rho*velocity^2); %L = 1/2 * rho * v^2 * S * Cl
test_Cd(j==rpms) = (2*drag)/(h(i)*wingspan*chordlength*rho*velocity^2); %D = 1/2 * rho * v^2 * A * Cd
test_Re(j==rpms) = velocity*chordlength/v; %Re = V * c / v
end
Cl(i) = mean(test_Cl);
Cd(i) = mean(test_Cd);
Re(i) = mean(test_Re);
fprintf('Cl = %4.2f, Cd = %4.2f, Re = %4.2f\n',Cl(i),Cd(i),Re(i));
RPM(rpms(1));
end
fprintf('Measurements Complete, Plotting Data and Saving Variables\n');
RPM(0);

% Resultant Force and L/D Expressions Dependent on Angle of Attack
R = (w.*Cl).^2 + (h.*Cd).^2; %Resultant Force is fully equal to R = 1/2 * rho * v^2 * l * c * (w^2 * Cl^2 + h^2 * Cd^2)^0.5
LD = (w.*Cl)./(h.*Cd); %Lift Drag Ratio simplified from Lift and Drag expressions

% Plotting Standardized Resultant Force and L/D Ratio against Angle of Attack
f(1) = figure('Name',strcat(string(NACA_airfoil), '_1',string(round(circle_x*100))),'% Standardized Resultant Force Compared to L/D Ratio','NumberTitle','off');
hold on
yyaxis left
plot(a,R)
t1 = title('Airfoil Standardized Resultant Force Compared to Lift/Drag Ratio as Functions of \alpha');
t2 = xlabel('Angle of Attack $\alpha$ (degrees)','Interpreter','latex');
t3 = ylabel('Standardized Resultant Force $w^2 \frac{C_L^2}{2} + h^2 \frac{C_D^2}{2}$','Interpreter','latex');
yyaxis right
plot(a,LD)
t4 = ylabel('Lift/Drag Ratio $\frac{wC_L}{hC_D}$','Interpreter','latex');
t1.FontSize = 15; t2.FontSize = 15; t3.FontSize = 15; t4.FontSize = 15;
grid on
grid minor
hold off

% Plotting Lift and Drag Coefficients against Angle of Attack
f(2) = figure('Name',strcat(string(NACA_airfoil), '_2',string(round(circle_x*100))),'% Lift and Drag Coefficients','NumberTitle','off');
hold on
yyaxis left
plot(a,Cl)
t5 = title('Lift and Drag Coefficients as Functions of $\alpha$');
t6 = xlabel('Angle of Attack $\alpha$ (degrees)','Interpreter','latex');
t7 = ylabel('Lift Coefficient (Cl)','Interpreter','latex');
yyaxis right
plot(a,Cd)
t8 = ylabel('Drag Coefficient (Cd)','Interpreter','latex');
t5.FontSize = 15; t6.FontSize = 15; t7.FontSize = 15; t8.FontSize = 15;
grid on
grid minor
hold off

% Saving Data and Plots
filename = strcat(string(NACA_airfoil), '_ClCd');
save(strcat(filename,'.mat'),'a','Cl','Cd','Re','w','h');
savefig(f,strcat(filename,'.fig'));
end
Appendix A.5: Inverting_Support.m

Numerical Process to Find Region of Airfoil Surface to Attach Interior Supporting Fabric to Maintain Airfoil Shape After Inversion

Josh Vandervelde, July 5, 2022, Summer 2022 Engineering Research

Problem: Find all points along top surface of airfoil whose distance to a support (along x axis, thus symmetric for inversion) is greater than the distance to the same support when the airfoil surface is pulled taut before full inversion and stopping the inversion process.

clear; % redo after centering airfoil

% User Input of X Location of Support
xS = 0.25; % x position of support beam center between 0 and 1
yS = 0.030; % y position of support beam center, measured by chord
rS = 0.394/2; % radius of support beams
scale = 24; % chord length

% Loading Variables from Other NACA Inversion MATLAB Functions
NACA_airfoil = 6412; % NACA airfoil for inversion as of Summer 2021 inverting_x = 0.1; % leading edge circle is centered 10% along the chordlength of the original 6412 airfoil
[xU,yU,xL,yL,circle] = NACAleadingcircle(NACA_airfoil,circle_x,0); % only require upper surface coords and plot close all; % get rid of this plot

% Rotate, Translate, and Scale Airfoil to Have Chord Length 1 Along X-Axis on the Origin
airfoil = [xU,flip(xL);yU,flip(yL)];
[chord,min_ind] = max((airfoil(1,:)-airfoil(1,min_ind)).^2 + (airfoil(2,:)-airfoil(2,min_ind)).^2); % find maximum difference
theta = atan((airfoil(2,min_ind))/airfoil(1,min_ind)); % angle of chord relative to x-axis

r = circle(3)/chord; xC = r; yC = 0; % radius and circle center coordinates of new circle
xT = xC; yT = yC + r; % coordinates of approximate tangency point of fabric (at top of circle)

% Find Coordinates of Upper Airfoil Surface
indU = airfoil(2,:) >= 0; % find indices of airfoil coordinates with a y value greater than or equal to zero
x = airfoil(1,indU); y = airfoil(2,indU); % gather upper airfoil coordinates
x = x(1:10:end); y = y(1:10:end); % take every tenth element for test case

% Finding points along surface that will end closer to support than beginning with
n = length(x);
p = sin(theta).*cos(theta); % rotates airfoil to align chord with x-axis

xF = p.*xT./((xT.^2 + yT.^2).^0.5); % pathlength from points to trailing
yF = p.*yT./((xT.^2 + yT.^2).^0.5); % final x coordinates of original points after inversion
indices = (x-xS).^2 + (y-yS).^2 >= (xF-xS).^2 + (yF-yF).^2; % logical array of points that end closer to support (after inversion)

% Create Circle for Leading Edge
thetac = linspace(0,pi,1000);
xLC = cos(thetac)*rS + scale*xS;
yLC = sin(thetac)*rS;

% Create Circles for Middle Beams
thetac = linspace(0,2*pi,1000);
xMC = cos(thetac)*rS + scale*xS;
yMC1 = sin(thetac)*rS + scale*yS;
yMC2 = sin(thetac)*rS - scale*yS;

% Create Circle for Trailing Edge
xTC = sin(thetac)*rS + 23.25;
yTC = sin(thetac)*rS;

% Plot Section of Airfoil That the Support Can Safely Attach To
f1 = figure('Name','Support Beams and Fabric on NACA 6412 10% Airfoil','NumberTitle','off');
plot(scale*airfoil(1,:),scale*airfoil(2,:),'
',scale*x(indices),scale*y(indices),'
',xLC,yLC, 'k',xMC,yMC1, 'k',xMC,yMC2, 'k',xLC,yLC, 'k',xTC,yTC, 'k');
title('Support Beams and Fabric on NACA 6412 10% Airfoil');
xlabel('Chord Length [in]');
ylabel('Thickness [in]');
axis equal; box on; grid on; grid minor;

% Deflection for 0.394"x0.288" support beams at 7.5ft with a force applied uniformly
w = 0.1; % uniform force per length (Newtons/meter) (0.22 lb/ft)
L = 7.5*3.048; % length of wing [m] (7.5ft)
E = 134e9; % Modulus of Elasticity carbon fiber tubes [Pa]
OD = 0.0100076; % outer diameter of tubes [m] (0.394"
ID = 0.0073152; % inner diameter of tubes [m] (0.288"

h = (ys*scale)*0.0254; % height from centroid to middle beams
Ixx = 0.75*pi*(OD/2)^4 - (ID/2)^4; % moment of inertia of flat beams
Ixz = pi*(OD/2)^4 - (ID/2)^4 + (pi*h)*(OD/2)^2 - (ID/2)^2; % moment of inertia of stacked beams
\[ d_1 = \frac{(1/0.0254)*(w*L^4)}{(8*E*I_{x1})}; \text{ deflection of flat beams [in]} \]

\[ d_2 = \frac{(1/0.0254)*(w*L^4)}{(8*E*I_{x2})}; \text{ deflection of stacked beams [in]} \]

% Plot Deflection vs Applied Weight
fl = figure('Name','Deflection vs. Uniform Wing Force for Support Placement','NumberTitle','off');
yyaxis left
plot(w,d1,'-');
xlabel('Applied Force per Length [N/m]');
ylabel('Wingtip Deflection [in]');
yyaxis right
plot(w,d2,'-');
ylim([0,1.5*max(d2)]); % visually separate the right and left axis linear plots
ylabel('Wingtip Deflection [in]');
title('Deflection vs. Uniform Force Distribution for Supports on a 7.5ft Wing');
legend('Original Design: One Middle Beam','New Design: Two Stacked Middle Beams','Location','northwest');
box on; grid on; grid minor;