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Teaching Time-Inconsistency Consistently

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by Mark Kuperberg*

Abstract

This paper places Kydland and Prescott's classic analysis of time inconsistency in a fuller context by examining their result in five different economic environments. Pedagogically, the paper uses a consistent mathematical treatment of the subject throughout. To understand the motivations of the central bank, the paper begins with the environment that Kydland-Prescott successfully criticized wherein the public has static inflationary expectations and the central bank assumes that they do. The main sections of the paper go on to analyze time inconsistency in four alternative environments defined by the belief structure of the central bank with respect to the nature of the economy and the mechanism by which the public forms its inflationary expectations. It is shown that the inconsistency result holds when the central bank does not understand the natural rate hypothesis and does not believe that the public forms their inflationary expectations rationally. The inconsistency result, however, does not hold in the other cases.

Keywords: time inconsistency, time consistency, monetary policy, inflation, rational expectations

JEL Codes: A22, B22, E02, E58, E61

Kydland and Prescott's analysis of time inconsistency in their article, "Rules Rather than Discretion: The Inconsistency of Optimal Plans" (1977), is a standard fixture in any advanced macroeconomics course. In addition to being one of the two contributions cited for their Nobel Prize, the article has launched a thousand papers that focus on the time inconsistency problem. The popularity of Kydland and Prescott's paper derives from two sources:

1) Many issues can be formulated in terms of dynamic inconsistency, and the Kydland-Prescott analysis accords with how economists think people behave in such situations, and
2) There is a general belief that their central example of policymaking and excessive inflation is both empirically relevant and arises under fairly mild assumptions.

This paper examines the second of these points and places the Kydland-Prescott analysis of dynamic inconsistency and inflation in a fuller, and mathematically consistent, context.

I. Case 0

Case 0 is presented not because it is a realistic description of the economy, but because it illustrates what the central bank is trying to achieve. In this Case, the central bank seeks to minimize the following loss function subject to the standard Phillips Curve and its belief that the public has static inflationary expectations:

Loss Function: $L(u,\pi)=u^2+\gamma(\pi-\pi^*)^2$
s.t. $\pi=\pi^e-\alpha(u-u_n)$

where:
γ is the relative regret that the central bank feels in missing its inflation target versus missing its unemployment target,
π^* is the central banker's inflation target,
π^e is expected inflation,
α is the slope of the short run Phillips curve, and
u_n is the natural rate of unemployment.

It is assumed that the public believes that the central bank will try to hit its announced inflation

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target despite the fact that the bank's loss function may dictate otherwise. This is the sense in which the public does not have rational expectations. All that is necessary for the central bank to achieve its goal is for inflationary expectations to be static or slow moving, but the algebra is simplest with the assumption that $\pi^e = \pi^*$. Given these expectations, the central bank's optimization problem is to choose π so as to minimize:

$$L(\pi) = \left[u_n - \frac{\pi - \pi^*}{\alpha}\right]^2 + \gamma(\pi - \pi^*)^2 \quad (1)$$

where the Phillips Curve has been substituted into the Loss Function to eliminate u^1, and π^e equals π^*. This yields the first order condition:

$$\frac{dL(\pi)}{d\pi} = 2\left[u_n - \frac{\pi - \pi^*}{\alpha}\right]\left[\frac{-1}{\alpha}\right] + 2\gamma(\pi - \pi^*) = 0$$

The solution for the inflation rate is:

$$\pi = \pi^* + u_n\left[\frac{\alpha}{1 + \alpha^2\gamma}\right]$$

And the solution for unemployment is:

$$u = \left[\frac{\alpha^2\gamma}{1 + \alpha^2\gamma}\right]u_n$$

Therefore, if the public believes that the central bank will try to hit its inflation target, the central bank will, in fact, choose an inflation rate above π^* with the result that the unemployment rate will be below u_n. Figure 1, which is a slight transformation of the figure in the original Kydland-Prescott article, illustrates this case where it is assumed that $\gamma = 1$, $\pi^* = 2\%$, $\alpha = .75$ and $u_n = 5\%$. The loss function is depicted as concentric semi-ovals with a global minimum at the black dot at [$u = 0$, $\pi = 2\%$]. The central bank is, however, constrained to be on the Phillips Curve that corresponds to expected inflation equal to π^* and therefore chooses point 0.

II. Case 1

Case 1: The central bank seeks to minimize the same loss function as in Case 0, but the public has rational expectations.

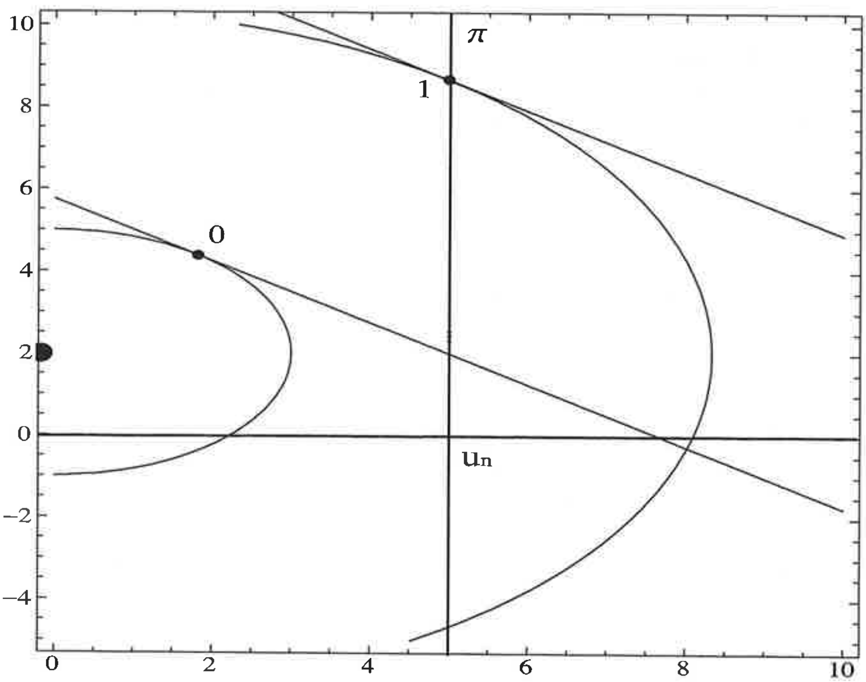

FIGURE 1. $\gamma = 1$.

The central message of Kydland and Prescott is that policymaking occurs in a dynamic environment where it is necessary to specify how the public is going to react to the policymaker's actions. In this case, the public is assumed to know the central bank's loss function and adjusts its inflationary expectations accordingly. π^e no longer equals π^* but rather π itself, since with knowledge of the central bank's loss function and no uncertainty in the model, the public can correctly calculate what inflation will be. The central bank, however, continues to assume that the public's inflationary expectations are equal to its inflation target. The loss function, where the Phillips Curve is once again used to eliminate u, is now:

$$L(\pi) = \left[u_n - \frac{\pi - \pi^e}{\alpha}\right]^2 + \gamma(\pi - \pi^*)^2 \quad (2)$$

The loss function in (2) differs from the loss function in (1) in that π^e no longer equals π^*. The first order condition becomes:

$$\frac{dL(\pi)}{d\pi} = 2\left[u_n - \frac{\pi - \pi^e}{\alpha}\right]\left[-\frac{1 - (d\pi^e/d\pi)}{\alpha}\right] + 2\gamma(\pi - \pi^*) = 0$$

When the central bank is minimizing its loss function in (2), it wrongly assumes that inflationary expectations are static so $d\pi^e/d\pi = 0$, but the public has rational expectations so $\pi^e = \pi$. Technically $d\pi^e/d\pi$ is **not** the derivative of π^e; it is the how the central bank believes the public's inflationary expectations are changing as the bank changes the money supply and the inflation rate. I could have introduced extra notation for this, but throughout the paper, this derivative will represent the central bank's belief as to how the public's inflationary expectations are changing, so this should create no confusion. The solution for inflation in this case is:

$$\pi = \pi^* + \frac{1}{\alpha\gamma} u_n$$

And the solution for unemployment is:

$$u = u_n$$

This is the classic Kydland-Prescott result. The central bank misunderstands the dynamic nature of the problem it is trying to solve and proceeds under the assumption that the public has static inflationary expectations. If the public did in fact have static expectations then the central bank would have been able to lower the unemployment rate at the cost of somewhat higher inflation (Case 0); but given that the public has rational expectations, the result of the central bank's actions is an inflation rate higher than its target while unemployment remains at the natural rate. This is depicted in Figure 1 as point 1 where $u = u_n$ and the slope of the loss function equals the slope of the Phillips Curve. The unemployment rate is no lower than it would have been had the central bank chosen $\pi = \pi^* = 2\%$, but, given the assumed values of the parameters, the inflation rate is 8.66%. The road to hell is paved with good intentions.

In their original article, Kydland-Prescott did not present the problem as a formal game between the policymaker and the public. The point they emphasized was that the optimal policy (point 0 in Figure 1) is not a consistent policy because the public's inflationary expectations do not equal actual inflation, and the consistent policy (point 1 in Figure 1) is not the central bank's optimal policy. Since then, however, so many papers have reframed the Kydland-Prescott result as a game (see Mankiw 2013) that many people think of time inconsistency solely in game theoretic terms. The standard one-shot game framework, however, is not identical to the analysis in this paper, because that framework makes the special assumption that the public moves first (ie. forms its expectations of inflation before the central bank takes action). Such a game results in the same outcome as Case 1 here, because the public anticipates that the central bank will inflate the economy beyond π^* and the resulting equilibrium will be Point 1. Parenthetically, it is conceivable that each individual member of the public might prefer Point 0 (with its misexpectations of inflation but lower unemployment) to Point 1, but as individuals they have no mechanism to mutually agree to underestimate inflation, and so they must take the rational expectations of others as given.

I will discuss the issue of credibility more generally at the conclusion of this paper. But briefly, the standard one-shot game framework places the nexus of the time inconsistency problem in the central bank's inability to credibly signal to the public that it will stick to its inflation target. This lack of credibility puts the central bank in a no-win situation once the public chooses π^e equal to the inflation rate at Point 1. One prominent partial solution to this problem is for the central bank to be more hawkish on inflation. In Figure 1, $\gamma = 1$ (the central bank cares as much about meeting its inflation target as it does about meeting its unemployment target). The bigger γ is, the more the central bank regrets missing its inflation target and the more hawkish it is about inflation. Figure 2 depicts the solution for $\gamma = 5$.

As can be seen in the Figure and in the Case 1 solution for inflation, a bigger γ results in a reduction in inflation as π at Point 1 moves closer to π^*. The central bank is hoping to reach point 0 with an inflation rate of 2.98%, but the economy ends up at point 1 with an inflation rate of 3.33%, which is considerably better than the 8.66% inflation rate found at Point 1 in Figure 1. The reason for this is that even though the public expects the central bank to "cheat" and raise inflation above its target, it expects a central bank that is hawkish on inflation to cheat less. This has led many authors to conclude that the central bank should have γ's that are significantly greater than 1 (Backus and Driffil 1985, Barro 1986). According to this analysis, a rational public should, in fact, choose a central banker who is more hawkish on inflation than they are themselves (Rogoff 1985).

III. Case 2

It is generally concluded that the Kydland-Prescott result emerges because the central bank ignores the fact that the public has rational expectations. But an equal driver of this result is that the central bank has a flawed objective function and fails to understand that the economy obeys the natural rate hypothesis. The bank has an unemployment target of zero and assumes it can maintain the unemployment rate below the natural rate without igniting increasing inflation. To see this, consider:

Case 2: The central bank seeks to minimize a loss function that is consistent with the natural rate hypothesis but continues to assume that the public has static inflationary expectations. The loss function is now:

$$L(u, \pi) = (u - u_n)^2 + \gamma(\pi - \pi^*)^2$$

which differs from Cases 0 and 1 in that the central bank suffers a loss if it drives u below u_n. Such a loss function, when combined with the Phillips Curve and the assumption that $\pi^e = \pi^*$, yields the following objective function to minimize:

$$L(\pi) = \left[-\frac{\pi - \pi^*}{\alpha}\right]^2 + \gamma(\pi - \pi^*)^2 \qquad (3)$$

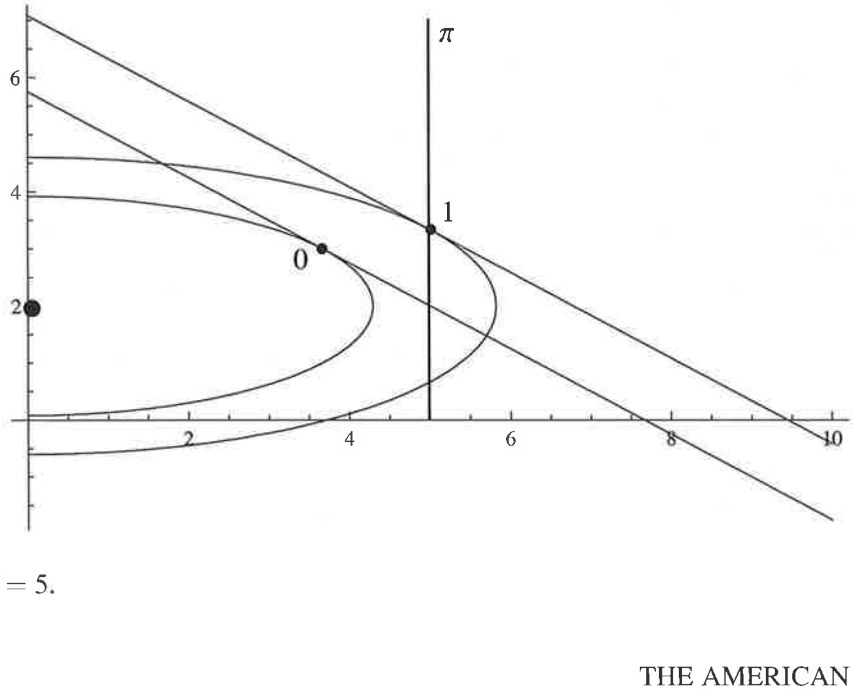

FIGURE 2. $\gamma = 5$.

which results in the first order condition:

$$\frac{dL(\pi)}{d\pi} = 2\left[-\frac{\pi - \pi^*}{\alpha}\right]\left[\frac{-1}{\alpha}\right] + 2\gamma(\pi - \pi^*) = 0$$

This time, the solution for the inflation rate is:

$$\pi = \pi^*$$

And the solution for unemployment is:

$$u = u_n$$

In this case, there is no dynamic inconsistency. So long as the central bank has an appropriate unemployment target, u_n, it will not attempt to lower the unemployment rate below u_n even though it can, given the public's static inflation expectations. In terms of Figures 1 and 2, the global minimum represented by the black dot moves to [$u = u_n$, $\pi = \pi^*$], the central bank has no incentive to deviate from this point.

The standard justifications for choosing an unemployment target below u_n are:

1) The central bank is politically motivated and myopic and seeks a short-term gain in lower unemployment even if it means permanently higher inflation. In this context, the Kydland-Prescott result is generally cited as an argument for central bank independence. While central bank myopia has certainly been present in particular places at particular times, the moral of Case 2 is that myopia by itself is not sufficient to generate excessive inflation. Also required is the central bank's belief that it is desirable to lower the unemployment rate below u_n.
2) The natural rate of unemployment is too high from the perspective of economic efficiency, so the central bank seeks to increase social welfare by lowering the unemployment rate below it. A suboptimal and excessively high natural rate of unemployment can occur for a host of microeconomic reasons, but the central bank, through standard monetary policy, cannot remedy any of these, and if the bank knows this, it is unlikely to try.

Whatever the central bank's motivations, its attempt to lower the unemployment rate below u_n is immediately self-defeating if the public has rational expectations and ultimately self-defeating under adaptive expectations. This self-defeating character of the Kydland-Prescott result highlights an asymmetry in the formulation of the problem. While the public understands the natural rate hypothesis and has rational expectations of the central bank's behavior, the central bank does not understand that u_n is the only long term equilibrium and has static expectations of the public's behavior. This second belief is justified in a one-shot game by the assumption that the public moves first, but this is a special case. In the more realistic case of repeated interactions between the central bank and the public, it is reasonable to assume that the central bank will realize that the public will alter its inflationary expectations as the central bank alters the inflation rate. This leads to Case 3.

IV. Case 3

The central bank seeks to minimize the same loss function as in Cases 0 and 1 subject to the standard Phillips Curve, but recognizes that the public has rational expectations. With the loss function as in Cases 0 and 1, the central bank, ignorant of the natural rate hypothesis, will try to drive the unemployment rate below u_n if it can. It is, however, constrained by its recognition that the public will react rationally to whatever it does. The objective function is:

$$L(\pi) = \left[u_n - \frac{\pi - \pi^e}{\alpha}\right]^2 + \gamma(\pi - \pi^*)^2 \qquad (4)$$

which results in the first order condition:

$$\frac{dL(\pi)}{d\pi} = 2\left[u_n - \frac{\pi - \pi^e}{\alpha}\right]\left[-\frac{1 - (d\pi^e/d\pi)}{\alpha}\right] + 2\gamma(\pi - \pi^*) = 0$$

The central bank's recognition that the public will react to the inflation rate it sets is embodied in the $d\pi^e/d\pi$ term, which unlike in Case 1, now equals 1. In the context of a one-shot game, taking the $d\pi^e/d\pi$ derivative is inappropriate because nothing the central bank does can alter the public's pre-formed rational expectations. In this Case, however, the derivative is designed as a short-hand way to capture the results of an entire literature that runs from Barro and Gordon (1983) to Li, Liu and Tian (2009) on the central bank's reputation in repeated games.

For example, Barro and Gordon analyze the following problem. Assume:

1) The central bank desires to lower the unemployment rate below u_n, as is true in Case 3 here, and
2) The central bank can fool the public and reach Point 0 in Figure 1 for one time period after which the public adjusts and the economy returns to Point 1, which is true not in Case 3 because $d\pi^e/d\pi = 1$.

Barro and Gordon then ask, "Is it in the interest of the central bank to generate a temporary boom at the cost of higher permanent inflation?" The answer they give is that generally the central bank will decide to stick to its inflation target and not try to fool the public.

In a like manner, the solution for the inflation rate in Case 3 is:

$$\pi = \pi^*$$

And the solution for unemployment is:

$$u = u_n$$

In this case, even though the central bank would like to lower u below u_n, its recognition of the public's rational expectations keeps it from attempting to do so. The intuition for this result comes directly from Barro and Gordon. If it is not in the interest of the central bank to create a temporary boom at the expense of permanently higher inflation, then it certainly will not be in the interest of the central bank to generate permanently higher inflation when, as in our Case 3, it cannot even create a temporary boom.

V. Case 4

For completeness, Case 4 is where the central bank recognizes both the natural rate hypothesis and the public's rational expectations. The math for this case is not shown because the two assumptions reinforce one another and result in the outcome: $\pi = \pi^*$ and $u = u_n$. In this case, the central bank has two reasons not to try to drive the unemployment rate below u_n, and we have seen in the previous cases that it only needs one.

VI. Summary

All of the results for Cases 1-4 are summarized in Table 1 below:

Alternatively, the results can be summarized in a generalized loss function:

General Loss Function: $L(u, \pi) = (u - u^*)^2 + \gamma(\pi - \pi^*)^2$

where u^* is the unemployment target: zero in Cases 0, 1 and 3 and u_n in Cases 2 and 4.

Combining this loss function with the Phillips Curve, taking the first order condition and solving for π, yields a generalized reaction function for the central bank:

$$\pi = \left[\frac{\alpha^2\gamma}{(1 - d\pi^e/d\pi) + \alpha^2\gamma}\right]\pi^* + \left[\frac{1 - d\pi^e/d\pi}{(1 - d\pi^e/d\pi) + \alpha^2\gamma}\right]\pi^e + \left[\frac{\alpha(1 - d\pi^e/d\pi)}{(1 - d\pi^e/d\pi) + \alpha^2\gamma}\right](u_n - u^*)$$

where $d\pi^e/d\pi$ equals zero in Cases 0, 1 and 2, and one in Cases 3 and 4.

TABLE 1.

Central Bank	Assumes the public has static expectations	Assumes the public has rational expectations
Does not understand the natural rate hypothesis	Case 1 Time inconsistency	Case 3 No time inconsistency
Understands the natural rate hypothesis	Case 2 No time inconsistency	Case 4 No time inconsistency

VII. What Constitutes Credibility?

As can be seen from Table 1, time inconsistency is more the special case than the general rule. This brings up the deep question of what constitutes credibility. Traditionally, a central bank's promise has not been considered credible unless the central bank genuinely hates missing its inflation target (a big γ) or there are big enough incentive mechanisms, such as performance contracts (Persson and Tabellini 1993), to encourage the central bank to stick to its inflation target. What Table 1 suggests is that a second set of credibility variables should be considered: what the central bank believes about the economy and what it believes about the mechanism by which the public forms its inflationary expectations. Only in the case where the central bank does not understand the natural rate hypothesis **and** does not believe that the public's inflationary expectations are formed rationally does time inconsistency arise. In all other cases, the central bank's belief about the economy or the public's expectations formation prevents it from trying to inflate the economy. Like the central bank's preferences with respect to missing its two targets (γ), its beliefs about the structure of the economy should factor into its credibility.

This brings us back to Case 0, and its historical relevance. Abstracting from the Oil Supply Shocks that generated inflation on their own, was the excess inflation in the United States in the 1970's due to the Federal Reserve's inability to make a credible commitment to a low inflation target or was it due to the Federal Reserve's misunderstanding of the structure of the economy? While the game theoretic commitment story may have wide applications in many areas of economics, a compelling case can be made that it was the Federal Reserve's lack of knowledge of the consequences of attempting to drive the unemployment rate below u_n that was responsible for the excessive inflation. In terms of the nomenclature in this paper, the Fed thought it was living in a Case 0 economy when in fact it was not. Likewise, a compelling case can be made that the low inflation rates that the U.S. economy has experienced since the 1970's are due to the Federal Reserve's improved understanding of the dynamics of inflation.

Notes

1. Alternatively, one can use the Phillips Curve as the constraint in a Lagrangian optimization problem, but in this case, the Lagrangian multiplier does not provide any additional useful information.

References

Backus, David and John Driffill. 1985. "Inflation and Reputation." *American Economic Review.* 75: 530–538.

Barro, Robert, and David B. Gordon. 1983. "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics.* 12: 101–121.

Barro, Robert. 1986. "Reputation in a Model of Monetary Policy with Incomplete Information." *Journal of Monetary Economics.* 17: 3–20.

Kydland, Finn E., and Edward C. Prescott. 1977. "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy.* 85: 473–492.

Li, Jingyuan, Yongming Liu and, Guoqiang Tian. 2009. "A reputation strategic model of monetary policy in continuous-time." *Journal of Macroeconomics.* 31: 523–533.

Mankiw, N. Gregory. 2013. *Macroeconomics*, New York, NY: Worth Publishers.

Persson, Torsten, and Guido Tabellini. 1993. "Designing Institutions for Monetary Stability." *Carneige-Rochester Conference Series on Public Policy.* 39: 53–84.

Rogoff, Kenneth. 1985. "The Optimal Degree of Commitment to an Intermediate Monetary Target." *Quarterly Journal of Economics.* 100: 1169–1189.