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Single-Equation Estimation of the Equilibrium Real Exchange Rate

*John Baffes, Ibrahim A. Elbadawi, and
Stephen A. O'Connell**

Estimating the degree of exchange rate misalignment remains one of the most challenging empirical problems in open-economy macroeconomics (Edwards (1989), Williamson 1994). A fundamental difficulty is that the equilibrium value of the real exchange rate is not observable. Standard theory tells us, however, that the equilibrium real exchange rate is a function of observable macroeconomic variables, and that the actual real exchange rate approaches the equilibrium rate over time (Edwards (1989), Devarajan, Lewis and Robinson (1993), Montiel (1997)). A recent strand of the empirical literature exploits these observations to develop a single-equation, time-series approach to estimating the equilibrium real exchange rate (Edwards (1989), Elbadawi and O'Connell (1990), Elbadawi (1994), Elbadawi and Soto 1994, 1995). Drawing on this earlier work, we outline an econometric methodology for estimating both the equilibrium real exchange rate and the degree of misalignment, and illustrate the methodology using annual data from Côte d'Ivoire and Burkina Faso.

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The procedure involves three steps. In the first step, the investigator identifies the long-run relationship to be estimated, adapting existing theory as necessary to key features of the country in question. This relationship is then embedded in a dynamic model whose long-run parameters are estimated in the second step, using techniques appropriate to the time-series characteristics of the data. In the third step, the investigator uses the estimated long-run parameters to calculate the equilibrium rate and the degree of misalignment under alternative assumptions regarding the sustainability of the fundamentals.

The chapter is organized accordingly. In the next section, we define the real exchange rate and derive an equilibrium relationship between the real exchange rate and a set of macroeconomic "fundamentals," including government spending patterns and the terms of trade. International credit constraints and changes in trade policy are potentially important features of the Côte d'Ivoire and Burkina Faso cases; and we show how these modify the list of fundamentals. We present the comparative statics and discuss the sources of short-run misalignment and dynamic adjustment. The section that follows, on motivating the single-equation approach, concludes the first step by embedding the long-run equilibrium in a single-equation, error-correction specification for the real exchange rate. This section provides a bridge to steps two and three by placing our approach in a broader stochastic context, discussing the relationship of our methodology to the standard PPP approach.

We then implement step two, starting in the section on estimation with an investigation of the time-series properties of the data. Côte d'Ivoire and Burkina Faso prove to be polar cases, with all variables nonstationary in Côte d'Ivoire and all variables (trend-) stationary in Burkina Faso. We focus particularly on Côte d'Ivoire, in which cointegration between the real exchange rate and its fundamentals opens up a menu of possible estimation approaches. We present the econometric results for both countries and discuss them in light of the existing empirical literature.

The section on calculating the equilibrium real exchange rate takes up the final step of the methodology. We discuss alternative ways of identifying "sustainable" values for the fundamentals and illustrate the alternatives for the cases at hand. Our preferred point estimates are based on counterfactual simulations for the fundamentals; these estimates suggest that by the end of the sample period (1993), Côte d'Ivoire was overvalued by roughly 30 percent while Burkina Faso had a small undervaluation.

In the end, of course, ongoing developments in time-series econometrics and the inevitable complexity of applied work leave us well short of attempting a "cookbook" in this chapter. Our more modest aim

is to provide sufficient detail—including pointers to the relevant literature and two extended illustrations—to encourage application of the methodology “in the field.” The final section concludes the chapter with an assessment of the practical value of the single-equation, time-series approach.

Step One: Modeling the Equilibrium Real Exchange Rate

The concept of the real exchange rate (RER) that has been most heavily used in analyses of external adjustment by developing countries is the domestic relative price of traded to nontraded goods (for example, Dornbusch 1983).¹ This is shown in equation 10.1:

$$(10.1) \quad RER \equiv e \equiv \frac{EP_T^W}{P_N}.$$

Although the world price of traded goods, P_T^W , is exogenous for a small country, the domestic price of nontraded goods is endogenous except over short periods of wage-price rigidity. The RER is therefore endogenous even under a predetermined nominal exchange rate. In this section we use a simplified model to illustrate the determination of the real exchange rate and derive an expression for its long-run equilibrium value. Since the relevant theory is well covered by Montiel in Chapter 5, we use his model as a basis for the discussion (see also Edwards (1989) and Rodriguez 1994).

The literature defines the long-run equilibrium real exchange rate as the rate that prevails when the economy is in internal and external balance for sustainable values of policy and exogenous variables. Internal balance holds when the markets for labor and nontraded goods clear. This occurs when the following equation 10.2 holds:

$$(10.2) \quad y_N(e, \xi) = c_N + g_N = (1 - \theta)ec + g_N, \quad \partial y_N / \partial e < 0, \quad \partial y_N / \partial \xi < 0$$

where y_N is the supply of nontraded goods under full employment, c is total private spending measured in traded goods, q is the share of this spending devoted to traded goods, and g_N is government spending on nontraded goods. The variable ξ is a differential productivity shock that raises the output of traded goods and lowers the output of nontraded

1. This is what Hinkle and Nsengiyumva call the “internal” real exchange rate in Chapters 2 and 3.

Figure 10.1 *Internal and External Balance*

Note: The *EB* schedule is drawn for steady-state values of the service account and transaction costs. A rise in e is a real depreciation.

goods at given relative prices (see below). Equation 10.2 is shown as the schedule *IB* in figure 10.1. Starting in a position of internal balance, a rise in private spending creates an excess demand for nontraded goods at the original real exchange rate. Restoration of equilibrium requires a real appreciation that switches supply toward nontraded goods and demand toward traded goods. A rise in government spending on nontraded goods shifts the *IB* schedule downward; a productivity shock in favor of traded goods shifts it upwards.

To define external balance, we begin with the current account surplus, which is given by equation 10.3:

$$(10.3) \quad \begin{aligned} \dot{f} &= b + z + rf = y_T(e, \xi) - g_T - (\theta + \phi)c + z + rf, \\ \partial y_T / \partial e &> 0, \quad \partial y_T / \partial \xi > 0 \end{aligned}$$

where f is total net foreign assets, b is the trade balance, z is net foreign grants received by the government, all measured in traded goods, and r is the real yield on foreign assets. The trade balance is the difference between domestic production of traded goods, y_T , and the sum of gov-

ernment (g_T) and private spending on these goods. The equation is standard except for the term ϕ , which measures the transactions costs associated with private spending. In Montiel's model of optimizing households, these costs motivate the holding of domestic money, which would otherwise be dominated in rate of return by foreign assets.² They are assumed to be incurred in the form of traded goods (at the rate ϕ per unit of spending) and therefore appear as an outflow in the trade balance.

External balance has been defined in various ways in the literature, with earlier approaches tending to focus directly on sustainable net capital flows and more recent work focusing on long-run stock equilibrium. We take the latter approach, following Montiel and others (for example, Khan and Lizondo (1987), Edwards (1989), and Rodriguez 1994). External balance therefore holds when the country's net creditor position in world financial markets has reached a steady-state equilibrium. We can solve for the combinations of private spending and the real exchange rate that are consistent with this notion of external balance by holding f at its steady-state level and setting the right-hand side of equation 10.3 to 0. This traces out a second relationship between the real exchange rate and private spending, labeled *EB* in figure 10.1. Starting at any point on this schedule, a rise in private spending generates a current account deficit at the original real exchange rate. To restore external balance, the real exchange rate must depreciate, switching demand toward nontraded goods and supply toward traded goods; the *EB* schedule is therefore upward-sloping. We will see below that this stock equilibrium concept of external balance is consistent with a sequence of "flow" restrictions on the trade balance when countries are rationed in the international financial market.

A fully specified macroeconomic model must also satisfy fiscal balance in the long run. Since the predetermined rate of crawl of the nominal exchange rate ties down seigniorage revenue as a function of predetermined money holdings (both measured in traded goods), some fiscal variable must ultimately adjust to guarantee fiscal balance. Government spending is being held fixed, so the adjustment falls to tax revenue. Montiel assumes that any incipient public sector deficit is financed continuously via lump-sum taxes or rebates. Fiscal balance therefore holds at each point in time in this model, with the required adjustments taking place behind the scenes. It is worth noting that if the exchange rate were freely floating rather than managed, the rate of crawl would become endogenous and choices regarding lump-sum taxation would help

2. Montiel assumes that transactions costs are a decreasing function of the ratio of money holdings to spending: $\phi = \phi(m/c)$, $\phi' < 0$.

tie down the long-run inflation rate; but in other respects the long-run schedules would be unchanged.³

The equilibrium real exchange rate, e^* , is given by the intersection of the *IB* and *EB* curves, which occurs at point 1 in the diagram. Setting the right-hand side of equation 10.3 to zero and combining this with equation 10.2, we obtain equation 10.4:

$$(10.4) \quad e^* = e^*(g_N, g_T, [r^* f^* + z], \phi^*, \xi)$$

- + - + -

where “*” superscripts denote steady-state values of endogenous variables and the signs below the equation are those of the corresponding partial derivatives of e^* . The signs of the partial derivatives in equation 10.4 are easily verified, either graphically or algebraically, using equations 10.2 and 10.3.

Montiel solves for the steady-state service account $r^* f^*$ by assuming that the country faces an upward-sloping supply curve of net external funds and that households optimize over an infinite horizon.⁴ Transactions costs per unit, f , are also endogenous; they depend on the ratio of money holdings to private spending and therefore on the nominal interest rate, which is the opportunity cost of holding domestic money. Since the nominal interest rate is tied down in the long run by the time preference rate and the domestic inflation rate, the final expression for the equilibrium real exchange rate takes the form

$$(10.5) \quad e^* = e^*(g_N, g_T, z, r_w, \pi_T, \xi)$$

- + - - + -

where r_w is the world real interest rate and π_T is the rate of inflation in the domestic price of traded goods.⁵ Note that the nominal exchange rate does not appear among the fundamentals in equation 10.5. This is

3. See Agenor and Montiel (1999) for an analysis of the effect of exchange rate regime (managed versus floating) on macroeconomic dynamics in a model similar to the one analyzed here. Note that taxes are assumed to be lump-sum and therefore nondistortionary in our model; otherwise tax rates would enter the long-run balance schedules.

4. The latter feature ties the domestic real interest rate to the time-preference rate in any steady state. Given r^* , the value of f^* is then determined uniquely by the external supply function.

5. Since $\pi_T = \pi_w + \pi_e$ where π_w is the world inflation rate and π_e is the rate of crawl of the nominal exchange rate, we can think of the latter two variables as among the fundamentals. Note also that we have suppressed the time-preference

because the underlying behavioral relationships are all homogeneous of degree 0 in nominal variables. A nominal devaluation therefore has at most a transitory effect on the real exchange rate.

Equation 10.5 emphasizes that the real exchange rate consistent with internal and external balance is a function of a set of exogenous and policy variables. In practical applications, this relationship between e^* and its macroeconomic "fundamentals" differentiates the modern approach to equilibrium real exchange rates from the earlier purchasing power parity (PPP) approach. Under PPP, the analyst would identify a reference period of internal and external balance and use the real exchange rate that prevailed during that period as an estimate of the equilibrium for other periods. Equation 10.5 implies that this is only legitimate if the fundamentals did not change between the reference and comparison periods. This criticism of the PPP approach is now widely accepted.⁶

The analysis underlying equation 10.5 can be readily modified to accommodate features that are important in particular applications. For our purposes, important extensions involve rationing of foreign credit, changes in the domestic relative price of traded goods, and short-run rigidities in domestic wages and prices. We discuss these extensions briefly in what follows.

Rationing of Foreign Credit

Equation 10.6 is derived under the assumption that the country faces an upward-sloping supply curve of external loans. The current account and trade balances are therefore endogenously determined at each moment by the saving and portfolio decisions of households. An extreme version of this view, more relevant for countries without access to commercial international borrowing on the margin, is that the country faces a binding credit ceiling (or equivalently, a floor on its international net

rate in writing equation 10.5. Finally, note that while the impact effect of a rise in the world real interest rate depends on whether the country is initially a net debtor or a net creditor, the steady-state *domestic* real interest rate is constant in this model (see previous footnote) and the long-run effect of a rise in r is independent of the country's (endogenous) net creditor position.

6. The period of macroeconomic balance used in PPP calculations can be a single year or a group of years; elsewhere in this volume these alternatives are referred to as the "PPP base-year approach" and the "PPP average or trend approach." In the section on the relationship of the PPP approach to the single-equation approach below we discuss further the distinction between these PPP approaches and our econometric approach.

creditor position). Since a binding credit ceiling shuts down the capital account and also determines net interest payments, the trade surplus becomes an exogenous function of aid flows both in the short run and in the long run provided the ceiling remains binding.⁷ Credit ceilings thereby generate a natural link between "stock equilibrium" concepts of external balance and "flow" approaches that define external balance as holding when the trade deficit is equal to exogenously given net resource transfers. With a binding credit ceiling equation 10.4 takes the simpler form (10.6):

$$(10.6) \quad e^* = e^*(g_N, g_T, b, \phi^* \xi, \dots)$$

- + + + -

In our empirical work below, we treat the trade surplus $b = -(rf + z)$ as one of the fundamentals, consistent with this interpretation.

The Terms of Trade, Trade Policy, and Productivity Differentials

The domestic relative price of exports and imports is given by equation 10.7:

$$(10.7) \quad \frac{P_X}{P_M} = \frac{\tau}{\eta}, \quad \tau \equiv \frac{P_X^W}{P_M^W}, \quad \eta \equiv \frac{1+t_M}{1-t_X}$$

where τ is the external terms of trade and η is a parameter summarizing the stance of domestic trade policy. If either τ or η changes over time, the analysis must be disaggregated to accommodate different real exchange rates for imports and exports—a point well emphasized elsewhere in this volume. The equilibrium real exchange rates for imports and exports can then be written as functions of the set of fundamentals identified above, along with τ and η . Since the real exchange rate for tradables is itself a geometrically weighted average of the real exchange rates for imports and exports, it will depend on the same set of fundamentals, and elasticities will depend on the relative weight (α) of imported goods in the tradables price index.⁸ Equation 10.6 then becomes equation 10.8:

7. The domestic real interest rate, in contrast, becomes endogenous. Movements in the domestic real interest rate reconcile private spending decisions with the exogenous credit constraint; and the spread between the domestic and foreign real interest rates captures the shadow price of the credit constraint.

8. Defining real exchange rates for imports and exports as $e_M = EP_M^W/P_N$ and $e_X = EP_X^W/P_N$ and the price index for traded goods as $P_T^W = (P_M^W)^\alpha (P_X^W)^{1-\alpha}$, the

$$(10.8) \quad e^* = e^*(g_N, g_T, b, \phi^*, \xi, \eta, \tau) \\ \quad \quad \quad - \quad + + + \quad - - ?$$

An improvement in the terms of trade increases national income measured in imported goods; this exerts a pure spending effect that raises the demand for all goods and appreciates the real exchange rate. This effect can in principle be overcome by substitution effects on the demand and supply sides, leading to an overall real depreciation. A tightening of trade policy appreciates the real exchange rate in the long run.

As outlined in the subsection on specifying an empirical model below, our fundamental task in this chapter will be to estimate the parameters of equation 10.8. To measure the real exchange rate we will use the ratio of foreign wholesale price indexes to domestic consumer prices (a measure of the "external RER," in the terminology of Chapter 1 of this volume). This has two important implications for the interpretation of equation 10.8. First, as discussed at length in Chapter 1, the external RER tends to move more closely with the internal real exchange rate for imports than with the internal real exchange rate for traded goods, e . While the magnitude of estimated elasticities will reflect this fact, the qualitative predictions indicated in equation 10.8 remain unchanged if the dependent variable is the internal real exchange rate for imports. This includes the ambiguity of the terms-of-trade effect, although there is a stronger tendency toward a real appreciation. The external RER has been widely used in empirical applications, and the spending effect has indeed proved dominant in most cases (for example, Edwards (1989), Elbadawi 1994).

The second implication of using an external RER measure is that the interpretation of the differential productivity shock ξ must be adjusted accordingly. A tendency for productivity to advance more rapidly in the production of traded goods than in nontraded goods is the basis of the celebrated Harrod-Balassa-Samuelson (HBS) explanation for why nontraded goods are systematically cheaper in poor countries than market exchange rates would suggest (see Obstfeld and Rogoff 1996). Equation 10.4, of course, focuses on the internal real exchange rate rather than on international comparisons of nontraded goods prices. A rise in ξ depreciates the internal equilibrium real exchange rate by increasing the relative output of traded goods. When using an external real exchange rate, however, the HBS effect comes into play. To the degree that

real exchange rate is $e = (e_M)^\alpha (e_X)^{1-\alpha}$. In our empirical work we use the ratio of foreign WPIs to the domestic CPI as our measure of the real exchange rate. As indicated elsewhere in this volume, this "external real exchange rate" tends to be a closer proxy to e_M than to the "internal real exchange rate for tradables."

differences between foreign and home productivity are concentrated in traded goods, these differences will show up in nontraded goods prices that are systematically higher in richer countries than purchasing power parity would suggest. This in turn means a more depreciated external real exchange rate for the home country, other things being equal. The sectoral shock ξ therefore captures the *difference* between trading partners and the home country in the relative productivity of labor in traded and nontraded goods. In our empirical work we use a ratio of foreign to domestic overall labor productivity as a proxy for ξ .

Nominal Rigidities and Short-Run Dynamics

In Montiel's model, domestic wages and prices are perfectly flexible and internal balance prevails continuously. If we consider the case of a binding credit ceiling, so that the trade balance is exogenous, we conclude that as long as changes in the fundamentals are permanent, the actual real exchange rate *never* deviates from its long-run equilibrium. This is apparent from the inspection of the internal and external balance schedules: with b tied down exogenously, e and c are free to adjust immediately to their new long-run equilibrium values when one of the fundamentals changes. This is illustrated in figure 10.2, in which we show the adjustment to an increase in the world real interest rate by a net debtor country facing a binding credit ceiling. For a given aid inflow, the rise in r_w increases the required trade surplus, shifting EB to the left (to EB') and depreciating the equilibrium real exchange rate. The adjustment from point 1 to point 2 is immediate; with a predetermined path for the nominal exchange rate the adjustment takes place through a fall in domestic prices and wages. Given wage-price flexibility, therefore, the binding credit constraint removes the model's only source of internal dynamics. The only remaining source of a divergence between the actual real exchange rate and its long-run equilibrium is a temporary change in one of the fundamentals.

If domestic wages and prices are sticky in the short run, a second important source of internal dynamics comes from disequilibrium in the labor market and the market for nontraded goods. As long as these markets eventually clear, the equilibrium real exchange rate is unaffected by the short-run nominal rigidity. But any shock that alters the equilibrium real exchange rate will now give rise to an adjustment process during which the actual real exchange rate will deviate from its new equilibrium. In figure 10.2, sticky wages and prices prevent the real exchange rate from moving to point 2 in the short run, so that output and spending take the burden of the external adjustment. The short-run equilibrium is at point 3, at which unemployment and inventory accumulation gradually push nominal wages and the prices of nontraded goods

Figure 10.2 Adjustment to an Increase in r_w (under a Binding Credit Constraint)

Note: A rise in r_w shifts EB upward to EB' . With flexible wages and prices, adjustment to the new long-run equilibrium at point 2 is immediate. With nominal rigidities, the economy jumps to point 3 and then converges gradually to point 2 along EB' . An upward movement is a depreciation of e .

down relative to the prices of traded goods. The real exchange rate depreciates over time, bringing the economy to point 2 in the long run. The process illustrated in figure 10.2 is often viewed as providing the primary role of nominal devaluation in macroeconomic adjustment: that of speeding an otherwise excessively slow and contractionary adjustment to an adverse external shock (Corden 1989).

As the foregoing observations suggest, the long-run relationship given by equation 10.8 is consistent with a variety of sources and patterns of short-run dynamics, including not only wage-price stickiness and gradual asset adjustment but also costs of labor mobility and other frictions not present above. In the section on specifying an empirical model we incorporate this feature by embedding equation 10.8 in a flexible specification of short-run dynamics.

Interpreting Real Exchange Rate Misalignment

In this chapter we follow Edwards (1989) and Montiel (1997) in using the term "misalignment" to denote the gap between e and e^* . There are

two important differences, however, between this descriptive use of the term and its more normative use in most policy discussions. The first is illustrated by our discussion of nominal rigidities. In the absence of nominal rigidities or other market imperfections, deviations between e and e^* are market-clearing responses to temporary movements in the fundamentals or to permanent movements that alter the long-run equilibrium level of net foreign assets. In such cases the gap between e and e^* has no clear normative significance, and in particular there is no presumption in favor of "corrective" policy intervention. The second difference stems from the observation that the real exchange rate may well be misaligned from a normative perspective even when the economy is in a steady-state equilibrium. Dollar (1992), for example, argues that African real exchange rates were systematically overvalued in the 1970s and 1980s, as a result of highly inward-looking trade regimes. In the theory developed here, the equilibrium real exchange rate is conditional on trade policies and other government interventions. Given these policy settings (whether socially optimal or not) misalignment is necessarily a temporary phenomenon, generated by short-run macroeconomic forces that prevent an immediate movement to the long-run equilibrium.⁹

Specifying an Empirical Model

In equation 10.8 we defined the equilibrium real exchange rate as the steady-state real exchange rate conditional on a vector of permanent values for the fundamentals. Given this structure, our task is to construct a time series for this unobserved variable (within sample and potentially out of sample), using data on the actual real exchange rate and fundamentals. As a first step we assume that the long-run relationship delivered by theory is linear in simple transformations (for example, logs) of the variables. Equation 10.8 therefore becomes equation 10.9:

$$(10.9) \quad \ln e_t^* = \beta' F_t^p$$

where e^* is the equilibrium real exchange rate and F^p the vector of permanent values for the fundamentals. Our task, therefore, is reduced to one of estimating the vector β of long-run "parameters of interest" and choosing a set of permanent values for the fundamentals appropriate to period t .

To estimate β we need an empirical model that is consistent with equation 10.9 but relates observable variables. We obtain such a model by translating into stochastic terms two straightforward and general fea-

9. For a more extensive discussion, see Chapter 5 by Montiel in Part II.

tures of the theory. The first is that equation 10.9 comes from a steady-state relationship between actual values of the real exchange rate and fundamentals. To capture this relationship we assume that the disturbance ω_t in the following equation 10.10

$$(10.10) \quad \ln e_t = \beta' F + \omega_t$$

is a mean-zero, stationary random variable.¹⁰

The second general feature of the theory is that the steady state is dynamically stable.¹¹ Shocks that cause the exchange rate to diverge from its (possibly new) equilibrium in the short run should produce eventual convergence to the relationship in equation 10.9 in the absence of new shocks (or equivalently, in conditional expectation). A specification that captures this notion while retaining consistency with both equation 10.9 and 10.10 is the general error-correction model expressed in equation 10.11:

$$(10.11) \quad \Delta \ln e_t = \alpha(\ln e_{t-1} - \beta' F_{t-1}) + \sum_{j=1}^p \mu_j \Delta \ln e_{t-j} + \sum_{j=0}^p \gamma_j' \Delta F_{t-j} + v_t,$$

where $F_t = [g_N, g_R, b, f, x, h, t]'$ is the vector of fundamentals and v_t is an independent and identically distributed, mean-zero, stationary random variable. Assuming that all variables are either stationary or I(1) (see below) in levels, equation 10.11 implies equation 10.10; and for $-2 < \alpha < 0$ the corresponding long-run equilibrium is stable.

Equation 10.11 embodies the central insight of the single-equation approach: that the equilibrium real exchange rate can be identified econometrically as *that unobserved function of the fundamentals towards which the actual real exchange rate gravitates over time* (Kaminsky (1987), Elbadawi (1994), Elbadawi and Soto (1994, 1995)). Note that in contrast to the long-run relationship, the short-run dynamics are not heavily restricted since equation 10.11 is just a re-parameterization of the unrestricted p^{th} -order autoregressive distributed lag (ADL) representation of $\ln e_t$, as shown in equation 10.12:

$$(10.12) \quad \ln e_t = \sum_{i=1}^p \mu_i^a \ln e_{t-i} + \sum_{j=0}^p \gamma_j^a F_{t-j} + v_t,$$

10. Note that equation 10.9 follows directly from equation 10.10 if $\ln e^*$ and F^P are interpreted as long-run conditional expectations of the relevant variables.

11. This does not rule out theoretical models that exhibit instability in certain directions (for example, rational expectations models); the key assumption is that the economy "chooses" a convergent path for given values of the fundamentals.

under the stability restriction $|\Sigma_j \mu_j^a| < 1$ and the assumption that the real exchange rate enters the long-run relationship.¹² For different parameter values, the unrestricted error-correction representation (equation 10.11) encompasses a wide variety of commonly used dynamic models (Hendry, Pagan, and Sargan (1984), Ericsson, Campos, and Tran 1991). This flexibility is an advantage, because although the dynamic structure of any *particular* theoretical model may place restrictions on the parameters in equation 10.11, these restrictions will depend on the nature of nominal and real rigidities, on whether households optimize or use rules of thumb, and on other model-dependent features that have little or no effect on the set of variables that enter the long-run equilibrium. With unrestricted dynamics, we allow the data maximum scope for determining their actual pattern, while retaining consistency with the long-run specification.

Much of our econometric work will take place in versions of equation 10.11. It is straightforward to incorporate variables that in theory do not belong among the long-run fundamentals, but that may affect the short-run dynamics. An example is the nominal exchange rate. Denoting such variables by a vector s , we would capture long-term effects by adding the term $\delta's$ inside the parentheses in equation 10.11 (allowing a test of the hypothesis $\delta = 0$) and short-term dynamics by adding $\Sigma_j \phi_j' \Delta s_{t-j}$ to the right-hand side. Equation 10.12 would then include the corresponding term $\Sigma_j (\phi_j^a)' \Delta s_{t-j}$, with $\delta = 0$ corresponding to a particular set of restrictions on the ADL parameter vectors $(\phi_j^a)'$. Equation 10.11 can also accommodate an intercept or deterministic trend and we can readily include dummy variables for potentially important exogenous events (for example, the Sahel drought of the early 1980s).

A Brief Detour: Motivating the Single-Equation Approach

Before moving to estimation we take a brief detour to place our approach in a broader context. This section can be skimmed without loss of continuity, although we encourage the reader to return to it when evaluating the overall methodology. We address three questions here. First, why restrict attention to equation 10.11 rather than studying the full joint

12. In terms of the ADL parameters, the adjustment speed α and long-run parameters β_j in the error-correction representation are given by $\alpha = \Sigma_j \mu_j^a$ (so that the stability restriction implies $\alpha < 0$) and $\beta_j' = -(\Sigma_j \gamma_j^a)/\alpha$. Note that we are also restricting the long-run impacts of the s variables to be zero.

distribution of the real exchange rate and its fundamentals? While information is generally lost by conditioning, we argue in the first subsection below that the alternative of systems-based estimation is unrealistic in small samples. Second, what is the role of econometric exogeneity in the single-equation approach? If the fundamentals are weakly exogenous, conditioning is without loss of relevant information and fully efficient estimation and inference can proceed in a single-equation setting. We introduce weak exogeneity in the first subsection (with technical details in appendix A). Weak exogeneity is testable. When it fails, the investigator faces a choice between systems estimation and instrumental variables. Strong and super-exogeneity also have natural applications in our approach, as outlined in the second subsection, on sustainable fundamentals and exogeneity requirements. Finally, a question of fundamental interest to practitioners: why go the econometric route at all, rather than relying on the standard PPP approach? The answer is more subtle than expected (see the last subsection, on the relationship to the PPP approach) and reveals the fundamental strengths and weaknesses of the two approaches.

Small Samples, Limited Information, and Weak Exogeneity

With reasonable generality the joint distribution of the real exchange rate, its fundamentals, and short-run variables can be represented by an n -variable vector autoregression (VAR) of finite order p , which in turn has a vector error-correction representation of the form shown in equation 10.13:

$$(10.13) \quad \Delta x_t = \Gamma x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t$$

where $x_t = [\ln e_t, Ft', s_t']'$ is the $nx1$ vector of variables and ε_t is the vector of reduced-form innovations (see appendix A). In general, efficient estimation of the parameters of equation 10.11 requires an analysis of the full joint distribution of the variables. A fundamental difficulty, however, is that sample sizes are likely to be very small. This is partly because the historical reach of developing-country data is typically short, and partly because models of the type considered here call for national accounts or fiscal data that are available only annually. For Côte d'Ivoire we have 29 annual observations; for Burkina Faso, 24. A general implication of small sample size is that the statistical properties of estimators may be poor and that testing procedures are likely to have low power. Existing Monte Carlo evidence can in some cases help discriminate

between alternative choices of estimator, but we will often have to make informal judgments about robustness to sample size. A second implication is that we are virtually forced into assuming that the parameters are constant over the sample. This assumption rules out structural changes that may in fact be present and, if incorrect, can produce misleading inferences about the stationarity properties of the data (see the section on the I[1] case below) and the values of the parameters.¹³

For our purposes, however, a more definitive effect of small samples is to limit the scope for systems-based estimation. The number of unknown parameters in the full joint distribution of the real exchange rate and its fundamentals rises roughly geometrically with the number of fundamentals and the lag length. With three or four variables among the fundamentals and fewer than 30 observations, this "curse of dimensionality" tends rapidly to overwhelm any attempt to estimate the full joint distribution. We will see below that the dimensionality problem is somewhat alleviated if the variables are nonstationary and cointegrated (and only the long-run parameters are of direct interest), but that even here the small sample size exerts a serious limitation on systems estimation. Our analysis will therefore generally take place in a single-equation context, in which we implicitly condition on the current values of at least a subset of the fundamentals and the lagged values of all variables.

Conditioning is at some potential cost, because efficient statistical inference regarding the parameters of interest—which may go beyond β to include the adjustment speed α and the short-run parameters μ_i and γ_j —generally requires analysis of the full joint distribution of $\ln e_t$, F_t and s_t . As shown by Engle, Hendry, and Richard (1983), however, fully efficient estimation and inference can take place conditional on the fundamentals if these variables are *weakly exogenous* for the parameters of interest. As outlined more fully in appendix A, weak exogeneity holds when the parameters of interest can be directly recovered from the

13. On the positive side, the shocks to developing-country data often appear to have high variance, thereby generating substantial variation over time; and temporal length of sample (as opposed to number of observations, which may increase because of a move from annual to quarterly data without lengthening the sample) has the same effect when the real exchange rate and its fundamentals are nonstationary variables. A relatively small sample may therefore contain substantial information, particularly regarding the long-run parameters. In the end, of course, this high variability is useful only if it can be parameterized in a sufficiently parsimonious manner; hence our caveat about ruling out structural changes.

distribution of the real exchange rate conditional on the fundamentals (and the past) *and* there are no cross-equation restrictions linking the parameters of this conditional model with those of the marginal model for the fundamentals. In this case the marginal distribution of the fundamentals holds no information of use to estimating the parameters of interest.

Weak exogeneity is testable (see appendix A), though generally at the cost of moving to systems estimation. Failure of weak exogeneity limits the scope for fully efficient conditional inference but need not undermine the ability to perform valid (though inefficient) inference in an essentially single-equation context. For regressions involving stationary variables, limited-information approaches such as two-stage least squares (or instrumental variables more generally) are available subject to sufficient identifying restrictions.¹⁴

For Côte d'Ivoire and Burkina Faso, the "small country" assumption suggests that variables such as the terms of trade and the foreign price level are determined outside the country.¹⁵ The same is true for the trade-weighted nominal exchange rate, since the CFA franc was pegged to the French franc at an unchanged parity throughout the sample. The trade balance is in this category if borrowing constraints are exogenous and binding. For these variables, weak exogeneity seems a reasonable assumption. Unfortunately, however, it is not guaranteed if behavior is affected by conditional expectations of these variables; for example, forecast errors will be jointly determined with the real exchange rate, potentially violating weak exogeneity. Variables such as government spending and the investment share may also be jointly determined with the contemporaneous real exchange rate. In what follows we test for weak exogeneity in the Côte d'Ivoire case and treat it as a maintained hypothesis for Burkina Faso.

14. When the the original variables are stationary, one option is to specify a dynamic simultaneous model or even a just-identified "structural VAR" along the lines of Bernanke (1986). This would require more identifying information than we are willing to impose, however. Moreover, system-based estimates that exploit this information are known to be less robust to mis-specification than limited-information approaches that ignore identifying information outside of the equation being estimated. Note also that limited information estimates can also support inference in cointegrated systems under failure of weak exogeneity, provided the equations being estimated involve only stationary variables and stationary combinations of nonstationary variables (that is, after the process Hendry 1995 calls "mapping to stationarity").

15. Côte d'Ivoire may well be large enough in the world cocoa market to affect its terms of trade.

Sustainable Fundamentals and Exogeneity Requirements

If we begin with equation 10.10, the equilibrium real exchange rate in equation 10.9 has a natural interpretation as the limit (as k goes to infinity) of a k -period-ahead conditional forecast of the real exchange rate. This suggests two broadly alternative ways of tying down the permanent values of the fundamentals. The first is using the sample information to generate long-run forecasts of the fundamentals conditional on information available in period t (or in some earlier period if t is out-of-sample). The second is combining theory and a priori information into a counterfactual simulation for the fundamentals. These correspond closely to the use of a single equation for conditional forecasting and "policy analysis." We argue below that the investigator will generally want to consider both alternatives. Here we briefly comment on the relevant exogeneity requirements (see Engle, Hendry and Richard 1983).

The requirements for valid single-equation forecasting and simulation generally go beyond those for valid estimation and inference. When using conditional forecasts of the fundamentals, the implicit assumption is that there is no feedback from the real exchange rate to the fundamentals. The appropriate concept is *strong exogeneity*, which combines weak exogeneity with lack of Granger causality from the real exchange rate to the fundamentals. Given weak exogeneity, strong exogeneity can be readily tested by determining whether lagged values of the real exchange rate enter the marginal model for the fundamentals.

When using counterfactual simulations of the fundamentals, the relevant issue is whether β can be treated as a constant in the face of shifts in the marginal distribution of the fundamentals. The problem here is the Lucas critique of econometric policy analysis: the counterfactual exercise implicitly alters the joint distribution of the fundamentals and the real exchange rate, thereby invalidating the original parameter estimates unless the corresponding parameters are invariant to the class of distributional shifts being considered. The appropriate concept in this case is *super-exogeneity*, which combines weak exogeneity with invariance of the parameters of interest to the class of distributional shifts under consideration. The invariance property is sensitive to the particular class of interventions under study and we will treat it as a maintained hypothesis rather than attempting formal testing.¹⁶

16. See Hendry (1995) and Ericsson and Irons (1995) on tests of super-exogeneity. Not surprisingly, such tests generally require intensive study of the relationship between the estimated equation and the associated reduced form for the fundamentals. One natural test (given weak exogeneity) relies on establishing parameter constancy in the estimated model given a sample break in the associated reduced form for the fundamentals.

Relationship to the PPP Approach

A hallmark of the PPP approach to equilibrium real exchange rates was the choice of a single equilibrium rate for all periods, without reference to movements in the fundamentals. The standard theory-based criticism, as embodied in our theoretical model, was that the notion of equilibrium delivers a *relationship* between the real exchange rate and fundamentals, not a single value for the real exchange rate. Since the fundamentals are themselves time-varying, this criticism has often been summarized in the claim that the equilibrium real exchange rate should move over time.

This way of stating the criticism, however, may miss the fundamental distinction between the PPP and econometric approaches. Consider the case in which the real exchange rate itself is stationary. Stationary variables have time-invariant means, implying that all movements away from the mean are ultimately temporary. In such a situation the best sample-based estimate of the equilibrium real exchange rate for any period is simply the sample mean. To put this another way, the quantity βF_t in equation 10.10 is the difference between two stationary variables and is therefore stationary, so that while the individual fundamentals may have permanent movements (that is, while they may be nonstationary), the relevant *function* of the fundamental—in our case, the long-run forecast of a linear combination of these fundamentals—*never* moves permanently. When forecasted at successively distant horizons, βF_{t+k} simply reverts to the mean of $\ln e_t$.¹⁷ An equilibrium relationship between the real exchange rate and other macroeconomic variables is therefore consistent with a time-invariant equilibrium real exchange rate, or in the trend-stationary case, with a deterministically trending equilibrium.

The more fundamental distinction between the two approaches resides in their contrasting use of sample and a priori information. The PPP approach requires a set of judgments that are informed both by theory and data but that remain largely implicit and a priori from an econometric perspective. The econometric approach, in contrast, uses theory sparingly but powerfully to extract information about the equilibrium real exchange rate from the entire data sample. A priori information becomes relevant when the analyst is interested in counterfactual simulations for the fundamentals, but such information is combined with

17. Estimation of long-run parameters appears superfluous in this case. The investigator will typically be interested, however, not only in a good conditional forecast of the real exchange rate but also in various characteristics of the short-run dynamics. Uncovering the relevant parameter values requires estimation even in the stationary case. Moreover, estimates of β are also required to apply counterfactual simulations for the fundamentals.

the sample information (used to estimate the parameters) in a restricted and transparent manner.

The econometric approach has clear advantages in reasonably large samples, where the high quality of the sample information should outweigh the loss of potentially sophisticated but implicit judgments central to the PPP approach. To give the PPP approach its due, however, we consider a problem that is peculiar to samples that are not necessarily small but are short in duration. We have just pointed out that in the stationary case, the sample mean provides a natural estimator of the long-run equilibrium real exchange rate. This implies, however, that the average misalignment within the sample is constrained to be zero. A similar though not identical outcome will tend to prevail in the nonstationary case: although the equilibrium rate itself is time-varying in this case, an important test of empirical success is that the equilibrium error is stationary. The resulting estimates of misalignment will then also tend to have a mean near zero if data-based forecasts for the fundamentals are used.

In other words, the econometric methodology tends by construction—except when counterfactual simulations of the fundamentals are used—to deliver an average misalignment of zero within the sample. This is in strong contrast to the PPP approach, which embodies no such restriction. In large samples, the restriction of a near-zero average misalignment is an unambiguous virtue, since it imposes the structure required to uncover the long-run parameters. But there may be severe problems in small samples, particularly if adjustment speeds are slow. Côte d'Ivoire's real exchange rate, for example, is thought by some to have been substantially overvalued for much of the post-WWII period. *Our methodology, when applied using data-based permanent values for the fundamentals, is essentially incapable of reproducing this finding.*

One response to this short-sample difficulty is to "rebase" the fitted equilibrium real exchange rates ex post by simply shifting their mean; this preserves their rates of change while altering the estimated degrees of misalignment. Despite its obvious appeal, however, rebasing has two important shortcomings. First, it leans very heavily on loosely structured a priori information, a feature of the PPP approach that the present approach is trying to avoid.¹⁸ Second, it embodies an implicit assumption

18. There is a sense in which rebasing can be characterized as imposing impossibly *tight* prior information: if the equilibrium rate is estimated via a static regression, rebasing by x percent is equivalent to imposing exact prior information of the form that the average degree of misalignment in the sample is x (since minimizing the sum of squared residuals subject to this constraint produces a shift only in the constant term)! There may be ways of making rebasing more palatable, however, without assuming either loosely structured or tightly structured

of super-exogeneity with respect to potentially substantial and largely implicit interventions in the marginal distribution of the fundamentals. Our use of counterfactual simulations for the individual fundamentals is a close cousin to the rebasing approach, but has the advantages of greater structure and transparency and, in particular, of exploiting the maintained super-exogeneity assumption more fully.

Viewed in this light, the PPP approach can be reinterpreted not primarily as an assumption that the equilibrium rate is a constant, but rather as an assumption that when samples are short and super-exogeneity *fails*, loosely structured a priori information (for example, "the economy was in internal and external balance in 1985") is of greater value to the policy analyst than the information contained in the sample distribution of the real exchange rate and fundamentals—even when the latter is combined with structured a priori information about the fundamentals.

Step Two: Estimation

Steps Two and Three involve estimating the long-run parameters in equation 10.11 and combining them with sustainable fundamentals to calculate the equilibrium real exchange rate. In what follows we outline these steps in detail and illustrate their implementation using data from Côte d'Ivoire and Burkina Faso. We begin with a discussion of the data and an investigation of the stationarity properties of the variables. Next we provide a brief overview of econometric considerations in the nonstationary and stationary cases. We end this section by presenting the econometric results.

The Data

As always in applied work, the documentation of definitions and sources of data (provided in appendix B) is fundamentally a list of compromises. We begin here with the real exchange rate, the measurement of which is treated comprehensively in Part I of this volume. We followed the bulk of the literature in using an external real exchange rate, the numerator of which is a trade-weighted average of foreign wholesale prices converted

omniscience. Elbadawi and Soto (1995), for example, determine time-varying sustainable values for the fundamentals and then let the data "choose" the reference period by identifying the quartile of sample years in which the vector of fundamentals is "closest" to the vector of permanent values. The entire set of equilibrium rates is then rebased to make the average misalignment in these years zero. A potential drawback is that this approach ignores misalignment associated with slow error correction (see equation 10.15 below).

at official exchange rates. An internal real exchange rate would have been preferable on theoretical grounds, and in the case of Côte d'Ivoire we experimented extensively with measures of the internal real exchange rate and also with the use of black market exchange rates in converting Ghanaian and Nigerian prices. The results were not robust across alternative measures, and those presented here are the strongest of the lot in terms of the plausibility of long-run parameters and the evidence of cointegration (see below). We return to the issue of robustness in our concluding section.¹⁹

To capture a possible Harrod-Balassa-Samuelson (HBS) effect associated with the use of an external real exchange rate, we used internationally comparable real GDP data to construct the ratio of real GDP per worker in the OECD to real GDP per worker in the home country. Since average labor productivity may be highly sensitive to demand-side influences in the short run, we used a three-year-lagged moving average of this ratio (with linearly declining weights). This variable performed poorly in preliminary stages for Côte d'Ivoire, however, and we dropped it for that country while retaining it for Burkina Faso.

Data limitations forced two further compromises worth discussing here. The first is that we were unable to separate government spending into traded and nontraded goods. Data were available, however, for the share of government consumption and investment in total spending, and we used these to proxy for the composition of spending. A rise in government spending appreciates the real exchange rate if government spending is more intensive in domestic goods than is private spending. A rise in the share of investment in aggregate spending is likely to shift spending toward traded goods, other things equal, given the high import content of investment, and therefore to depreciate the real exchange rate. The second compromise is that in lieu of direct measures of the stance of trade policy, we had to construct proxies for this variable. It is common in this literature to use various ratios of trade to GDP, on the argument that a more liberal trade regime, other things being equal, means higher trade volumes. We experimented with three such proxies: the ratio of current imports to current GDP, the ratio of constant-price imports to constant-price GDP, and the ratio of total trade (imports plus

19. Annual data were used for all variables. Although monthly data were available for the external RER measure used, data for most of its fundamental determinants were available only on an annual basis. Moreover, use of monthly data for the RER would have introduced seasonal and other transitory fluctuations, increasing the noise in the time series without improving the accuracy of the statistical estimates. (For a further discussion of this point, see MacDonald 1995).

exports) to constant-price GDP. All three performed adequately for Burkina Faso, but in Côte d'Ivoire the ratio of current imports to current GDP was clearly superior to the other proxies. We therefore retained only this proxy in the analysis reported here. For the case of Côte d'Ivoire we also included a drought dummy variable that takes the value of one for 1983 and 1984 and zero otherwise. Since agriculture is primarily a traded-goods sector, this supply shock should depreciate the real exchange rate.

Determining the Order of Integration

Macroeconomic data often appear to possess a stochastic trend that can be removed by differencing once. Since the presence of such a trend influences the statistical behavior of alternative estimators, a key preliminary step is to determine the order of integration of the data. Variables that are best characterized as nonstationary in levels but stationary after differencing once are integrated of order 1, or $I(1)$. Other variables may be stationary ($I(0)$) or trend-stationary (that is, $I(0)$ after removing a deterministic trend component), or they may require repeated differencing before achieving stationarity ($I(d), d > 1$). These properties can readily be revealed using standard tests for the presence of a unit root.²⁰ The appropriate unit-root tests are well known; in our applications we use the Dickey-Fuller (DF), augmented Dickey-Fuller (ADF), and Phillips-Perron (PP) tests. Although there are concerns about the low power of these tests against stationary but persistent alternatives, the ADF test appears to perform satisfactorily on this score even when (as in our case) the number of observations is small (Hamilton 1994). We also supplement the unit-root tests with variance ratio tests (Cochrane 1988); these tests exploit the fact that the variances of conditional forecasts explode for nonstationary series and converge for stationary series as the forecast horizon grows.

20. Hamilton (1994) emphasizes the difficulty of distinguishing truly nonstationary processes from processes that are stationary but persistent. The problem is that the finite-sample autocovariances of any nonstationary series can be reproduced arbitrarily closely by those of a suitably persistent stationary series. The usual tradeoff between consistency and efficiency is therefore present even at this preliminary stage. If we correctly characterize the order of integration, we gain efficiency in estimation and inference by applying the appropriate estimation technique; but a misclassification typically means that these techniques will deliver inconsistent estimates or standard errors. Unfortunately the alternatives are non-nested and we see no generally robust way of proceeding in marginal cases. Hamilton (p. 447) suggests comparing estimates obtained under alternative classifications; if they differ widely the investigator may sometimes see ancillary statistical or other grounds for preferring one over the other.

Table 10.1 Stationarity Statistics—Levels without and with Time Trend

	Côte d'Ivoire			Burkina Faso		
	DF	ADF	PP	DF	ADF	PP
<i>Levels without Time Trend</i>						
log(REER)	-0.59	-1.26	-1.89	-2.25	-4.25	-2.25
log(TOT)	-1.42	-1.54	-1.78	-1.95	-1.82	-1.87
RESGDP	-2.11	-2.57	-2.25	-3.84	-2.22	-4.07
log(OPEN1)	-1.06	-1.39	-1.42	-4.02	-3.04	-4.30
log(OPEN2)	-2.35	-1.99	-2.48	-3.23	-3.02	-3.35
log(OPEN3)	-2.52	-2.16	-2.69	-3.63	-2.99	-3.82
log(HBS3)	n.a.	n.a.	n.a.	-1.21	-2.05	-1.67
log(ISHARE)	-1.01	-0.78	-0.68	n.a.	n.a.	n.a.
<i>Levels with Time Trend</i>						
log(REER)	-1.83	-2.46	-2.09	-4.89	-2.76	-5.35
log(TOT)	-1.51	-1.56	-1.69	-2.30	-2.08	-2.34
RESGDP	-2.05	-2.50	-2.24	-4.27	-2.69	-4.64
log(OPEN1)	-1.02	-1.32	-1.29	-3.84	-2.94	-4.20
log(OPEN2)	-2.81	-2.30	-3.02	-3.12	-2.95	-3.31
log(OPEN3)	-2.47	-1.99	-2.72	-3.47	-2.91	-3.75
log(HBS3)	n.a.	n.a.	n.a.	-3.65	-3.75	-3.68
log(ISHARE)	-2.42	-2.19	-2.42	n.a.	n.a.	n.a.

Note: DF, ADF, and PP refer to Dickey-Fuller, augmented Dickey-Fuller, and Phillips-Peron stationarity statistics. The number of observations is 29 for Côte d'Ivoire and 24 for Burkina Faso. The variables are defined in appendix B (ISHARE is not available for Burkina Faso).

Source: Computed from data from sources listed in appendix B.

Table 10.1 shows the results of unit-root tests for all stochastic variables. Côte d'Ivoire and Burkina Faso represent two extremes. For Côte d'Ivoire, all three tests indicate nonstationarity for all variables. Moreover, we can reject the unit-root hypothesis for the first difference of the variables (not reported), so we conclude that these are $I(1)$ variables. For Burkina Faso, all variables appear to be trend-stationary, with the possible exception of the terms of trade, which is bordering on nonstationarity. Figures 10.3 and 10.4 provide some additional information in the form of variance ratio tests.²¹ These tests corroborate the unit-root

21. This ratio is defined as $(1/k)\text{Var}(X_t - X_{t-k})/\text{Var}(X_t - X_{t-1})$, where X_t is the variable of interest and k is the lag length (Cochrane, 1988).

Figure 10.3 *Variance Ratio Tests for Côte D'Ivoire*

Source: Computed from data from sources listed in appendix B.

tests, and for Burkina Faso's terms of trade the variance ratios decline at longer horizons, consistent with a persistent but stationary variable. We therefore proceed under the assumption that the terms of trade are stationary. In principle, of course, the vector $[\ln e_t, F_t', s_t']'$ may contain an arbitrary combination of $I(0)$ and $I(1)$ —or even $I(2)$ —variables. We focus our exposition, however, on the two cases represented by our examples.²²

22. Methods have recently been developed that allow consistent estimation and inference in regressions that involve mixtures of integrated processes. See Phillips (1995) and Phillips and Chang (1995).

Figure 10.4 *Variance Ratio Tests for Burkina Faso*

Source: Computed from data from sources listed in appendix B.

The I(1) Case

When the variables are all $I(1)$, as for Côte d'Ivoire, stationarity of the residual ω_t in equation 10.10 implies that the real exchange rate and its fundamentals are *cointegrated* (Granger 1981). This property is extremely useful econometrically, and a massive literature has developed in the wake of Engle and Granger (1987). As shown by Johansen (1988), cointegration is a restriction on the reduced form or VAR representation

of the joint distribution of the real exchange rate and its fundamentals, equation 10.13. If the number of linearly independent stationary combinations of the variables is r ($0 < r < n$), then the matrix Γ in equation 10.13 is of reduced rank $r < n$. We can then write $\Gamma = ab'$, where a and b are two nxr matrices of rank r .²³ The columns of b span the "cointegrating space" of stationary combinations of the x_{it} ; the rows of a give the weights with which these combinations enter the individual equations of the reduced form. Equation 10.13 becomes equation 10.14:

$$(10.14) \quad \Delta x_t = ab'x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t$$

where as before $x_t = [\ln e_t, F_t', s_t']'$ is the $nx1$ vector of variables in the system. Since the cointegrating vectors are identified only up to a normalization, we are free to impose r restrictions on the b matrix; for example, we might choose the normalization $b_{ii} = 1$, $i = 1, \dots, r$. We will restrict attention in this chapter to the case in which $r = 1$, so that there is a single cointegrating vector. The normalization on $\ln e_{t-1}$ (which assumes only that $\ln e_{t-1}$ actually enters the long-run relationship) then exactly identifies the remaining components of the cointegrating vector.²⁴ With a single cointegration vector, then, a and b are $nx1$ vectors of the form $a = [a_1, a_2]'$ and $b = [1, \beta']'$, where 1 is the scalar weight on the equilibrium error bx_{t-1} in the first row of equation 10.14. Note that if the st are truly short-run variables, their long-run coefficients will be zero.

23. Since each of the variables in x is either $I(0)$ or $I(1)$, all of the first differences in equation 10.14 are stationary. Stationarity of ε_t then implies that each row of Γx_{t-1} must also be stationary (since it is a linear combination of stationary variables) although the individual x_{it} 's are all nonstationary. This is accomplished if the rows of Γ induce stationary linear combinations of the nonstationary variables x_{it} ; hence the decomposition of Γ . Note that if there are n stationary combinations, then the individual x_{it} 's must all be stationary.

24. If there are multiple cointegrating relationships, normalization alone is insufficient to relate the long-run parameters uniquely to their counterparts in any particular economic theory—that is, to obtain interpretable parameter estimates. In addition we require further identifying restrictions (see, for example, Johansen and Juselius 1994). In this case the single-equation approach is likely to pick up a weighted combination of the cointegration vectors (Johansen, 1992). This lack of identification may not be highly damaging for forecasting purposes, but it raises a variety of issues that go beyond the scope of single-equation approaches. The closest counterpart to our approach in the $r > 1$ case is the "structural error correction model" of Boswijk (1995; discussed in Ericsson 1995), which is obtained by premultiplying equation 10.14 by a square matrix and then imposing a set of restrictions.

Determining the Cointegrating Rank

The cointegrating rank is a property of the full system, and a system estimator is required to test for it. Table 10.2 reports the results of Johansen's likelihood ratio tests for the cointegrating rank in Côte d'Ivoire. We use a lag length of one for the underlying VAR system; this is very restrictive even for annual data, but longer lag length leaves us with very few degrees of freedom. The null hypothesis for these tests is that the number of cointegrating vectors relating the n nonstationary variables is less than or equal to r (where $r < n$). Comparing the estimated likelihood ratios in column 2 to the asymptotic critical values in column 3, we see (row 1) that the hypothesis of no cointegration ($r = 0$) can be rejected in favor of at most one cointegrating vector. In row 2, the hypothesis of one vector cannot be rejected in favor of more than one. The asymptotic tests therefore indicate one cointegrating vector.

Likelihood ratio tests of cointegration are known to be sensitive to small-sample bias, tending to reject low values of r too often. In columns 3 and 5 we show a set of critical values that adjust for small-sample bias using a method suggested by Cheung and Lai (1993). Using these critical values it is difficult to distinguish between zero and one

Table 10.2 Johansen's Maximum Likelihood Test of Cointegration Rank for Côte d'Ivoire

		10 Percent Critical Value		5 Percent Critical Value	
	L-Max	Unadjusted	Adjusted	Unadjusted	Adjusted
With the drought dummy variable					
$r = 0$	45.01	36.35	48.34	39.43	52.44
$r \leq 1$	30.05	30.84	41.02	33.32	44.31
Without the drought dummy variable					
$r = 0$	32.65	30.84	39.17	33.32	42.32
$r \leq 1$	18.63	24.78	31.47	27.14	34.47

Note: The first row ($r = 0$) tests the null hypothesis of no cointegration; the second ($r \leq 1$) tests the null of at most one cointegration vector. The first column (L-Max) gives the estimated Johansen likelihood value in each case. The second and fourth columns give the 10 percent and 5 percent critical values taken from Osterwald-Lenum (1992, table 1.1). The third and fifth columns give the small-sample-adjusted critical values. The adjustment factor is calculated as $T/(T-nk)$, where T is the number of observations (28), n is the number of variables including the intercept (6) and drought dummy variable where included, and k is the number of lags (1). When the dummy is included (upper panel), the adjustment factor is 1.33; when it is excluded, this becomes 1.27. See Cheung and Lai (1993) for discussion of the adjustment factor.

Source: Computed from data from sources listed in appendix B.

cointegrating vector. We will proceed under the assumption that there is one vector, although we are marginally unable to reject the hypothesis of zero at the 10 percent level using the adjusted critical values.²⁵

Alternative Estimators

There are a number of potential approaches to estimating the cointegrating parameters. The simplest and earliest is the Engle-Granger (1987) "two-step" method, which applies OLS to a static regression relating the levels of the real exchange rate and its fundamentals (equation 10.10). Cointegration implies that the residuals from this regression are stationary, and this restriction provides a test for cointegration. Because of the dominance of the common stochastic trend, the estimates of β from the static regression are super-consistent, approaching the true parameters at a rate proportional to the sample size rather than the square root of the sample size; and they remain so even in the absence of weak exogeneity. In the second step, lagged residuals from the static regression are used in place of the equilibrium errors on the right-hand side of a reduced-form error-correction equation. Again OLS provides consistent estimates, this time of the adjustment speed α and short-run parameters of the error-correction specification.²⁶

While the Engle-Granger method is extremely simple to implement, the estimates of the cointegrating vector are biased in small samples. The degree of bias depends on the degree of persistence in the residual, suggesting that superior estimates might be obtained by accounting for the short-run dynamics (Banerjee and others 1993). We therefore also report OLS estimates of β taken directly from the error-correction specification (equation 10.9). These control for the short-run dynamics—which may be of interest themselves—and, like the static regression estimates,

25. We include the drought variable in the long-run relationship, on the grounds that it picks up a supply shock that is highly asymmetric between traded and nontraded goods. Unfortunately, the critical values of Dickey-Fuller tests and the majority of the tests used in the Johansen procedure are sensitive to the exact specification of deterministic variables in the cointegrating relationship. We do not attempt the Monte Carlo simulations that would be required to establish critical values for our case.

26. Engle and Granger (1987) demonstrated an equivalence between cointegration and error correction for nonstationary variables. In the nonstationary case, therefore, equation 10.10, which implies cointegration, also implies that the real exchange rate has a reduced-form error-correction representation—that is, one that is similar to equation 10.11 but with contemporaneous values of the fundamentals excluded. It is this reduced-form error-correction equation that is estimated in the second step of the Engle-Granger method.

remain consistent even with a failure of weak exogeneity.²⁷ Moreover, in line with our earlier discussion, a second and potentially decisive advantage emerges under weak exogeneity: estimates of β taken from the conditional error-correction model are equivalent to full-information maximum-likelihood estimates. They are therefore asymptotically efficient, and the t -ratios generated by OLS are asymptotically normal, allowing standard inference. This is in contrast to the static regression case, where the t -ratios have nonstandard distributions even asymptotically.

A third natural alternative is the Johansen (1988) procedure, which is a systems approach based on estimation of the full VAR in equation 10.13. The "curse of dimensionality" is a serious limitation here, however. Monte Carlo evidence suggests that the Johansen procedure deteriorates dramatically in small samples, generating estimates with "fat tails" (in other words, frequent outliers) and sometimes substantial mean bias. Moreover, the procedure is less robust than the single-equation alternatives to mis-specification of system parameters such as lag length and to practical features such as serial correlation in the equilibrium error (Hargreaves 1994). Because of these small-sample problems, we limit our use of the Johansen procedure to determination of the number of cointegration vectors and investigation of weak exogeneity, both of which are features of the entire system of equations 10.13. For estimation purposes we restrict attention to the single-equation methods.

The I(0) Case

In the case of Burkina Faso, we find that all variables are stationary in levels. We pointed out above that in this case, the long-run "equilibrium" value of $\ln e_t$, like that of any stationary variable, is simply its mean. A consistent and efficient estimator of the equilibrium real exchange rate is therefore the sample mean, corrected for any deterministic trend. This implies that the long-run parameters need not be estimated for the purpose of tying down the long-run equilibrium. If the fundamentals are super-exogenous with respect to these parameters, however, a structural shift in the marginal process generating the fundamentals (for example, a shift in the mean of F_t) will produce a corresponding change in the mean of $\ln e_t$, with the slope of the effect given by the associated long-run parameter. Moreover, the long-run parameters and the short-run dynamics may be of theoretical interest even in

27. A failure of weak exogeneity, however, means small-sample bias and invalid inference regarding the long-run parameters. Recall also that the conditions for weak exogeneity with respect to short-run parameters are stronger.

the absence of super-exogeneity; and the investigator may have a practical interest in generating short-to-medium-term conditional forecasts of the real exchange rate. For all of these reasons, we proceed with estimation in the stationary case, even though it is not strictly necessary for assessment of the long-run equilibrium.

The theory of specification and estimation in the stationary case is well developed and we will not review it here; see Hendry 1995. What is clear is that the existence of a long-run relationship no longer exerts the kind of statistical leverage that it does when the variables are individually nonstationary. This is apparent in equation 10.10 since all the dynamics have been pushed into the residual ω_t , which is therefore likely to be correlated with the right-hand side variables. OLS estimates of the static regression are therefore inconsistent in the $I(0)$ case, even though (as emphasized above) they are super-consistent when the variables are nonstationary and cointegrated.²⁸ The error-correction model addresses this problem to some degree by incorporating dynamics; but the contemporaneous values of the fundamentals still raise issues of predeterminedness. Lacking identifying information on equation 10.11, one way to obtain consistent estimates of the parameters in that equation is to use higher lags of the fundamentals as instruments.²⁹

Empirical Results

Tables 10.3 and 10.4 contain estimation results for Côte d'Ivoire while table 10.5 contains results for Burkina Faso. For Côte d'Ivoire, table 10.3 shows long-run parameters obtained from OLS regressions in levels (the first step of the Engle-Granger two-step method), using three alternative

28. The standard sufficient condition for consistency of OLS in the stationary case is that the right-hand side variables are *predetermined*—that is, that the residual is uncorrelated with contemporaneous and lagged right-hand side variables. In equation 10.10 the condition is $Cov(\omega_t, F_{j,t-k}) = 0$ for $k > 0$ and for each of the fundamentals F_j . In the stationary case, predeterminedness corresponds closely (but not exactly) to weak exogeneity (Engle, Hendry, and Richard 1983); Monfort and Rabemananjara 1990).

29. The lack of a clear statistical distinction between the individual and joint variation of the variables carries over to the conditions for weak exogeneity, which now make no general distinction between the short- and long-run parameters. A sufficient condition in the present limited information context (that is, in which identifying restrictions on the marginal model are not available) is that equation 10.11 and the marginal model form a block-recursive system (which guarantees predeterminedness and obviates the need for instrumental variables). We do not formally test for weak exogeneity in the $I(0)$ case (Burkina Faso), treating it instead as a maintained hypothesis where necessary (see Monfort and Rabemananjara 1990).

Table 10.3 Long-Run Parameter Estimates for Côte d'Ivoire Using Alternative Proxies for Openness

	OPEN1	OPEN2	OPEN3
Constant	-3.61 (-16.71)	-4.29 (-22.01)	-4.30 (-12.22)
log(TOT)	-0.04 (-3.03)	-0.16 (-1.06)	-0.15 (-0.94)
RESGDP	2.67 (5.49)	1.47 (3.25)	1.45 (3.71)
log(OPEN)	0.78 (3.68)	0.08 (0.34)	0.03 (0.12)
log(ISHARE)	0.27 (5.83)	0.31 (4.63)	0.30 (5.15)
D8384	0.22 (3.01)	0.30 (3.43)	0.30 (3.49)
Adjusted R ²	0.72	0.56	0.56
Q	14.32 (0.05)	13.80 (0.05)	14.21 (0.05)
DW	1.16	1.14	1.15
DF	-3.55	-3.31	-3.31
ADF	-3.54	-3.84	-3.89
PP	-3.61	-3.30	-3.29

Note: The numbers in parentheses are *t*-ratios (note that these have nonstandard distributions even asymptotically). The period of estimation is 1965–93. The three regressions use the alternative openness variables discussed in appendix B. The dependent variable is *log* (*REER*).

Source: Computed from data from sources listed in appendix B.

versions of the openness variable. There is strong evidence of cointegration in each case, as indicated by the unit-root tests applied to the estimated residuals: in each case the calculated values reject nonstationarity in favor of stationarity at standard levels.³⁰ Since the OPEN1 results are generally strongest, we use this variable in what follows. Except where otherwise noted, in the following discussion we focus on columns 1 and 3 of table 10.4 for Côte d'Ivoire and column 3 of table 10.5 for Burkina Faso. For Côte d'Ivoire, the selected columns

30. Note that the critical values for this test are more demanding than when testing for a unit root in a single variable, since the OLS estimation tends to induce stationarity in the residual.

Table 10.4 ECM Parameter Estimates for Côte d'Ivoire

	Two-Step ECM		Unrestricted ECM	
	OLS	IV	OLS	IV
Constant	5.60 (25.99)	5.69 (25.28)	5.54 (15.02)	5.53 (9.13)
Adjustment Speed				
$\log(\text{REER}_{t-1})$ or Error_{t-1}	-0.34 (-2.05)	-0.39 (-2.09)	-0.45 (-2.32)	-0.37 (-1.63)
Long-Run Parameters				
$\log(\text{TOT}_{t-1})$	-0.40 (-3.03)	-0.50 (-3.24)	-0.54 (-2.83)	-0.75 (-2.21)
RESGDP_{t-1}	2.67 (5.49)	2.81 (5.58)	2.60 (3.04)	1.53 (1.04)
$\log(\text{OPEN}_{t-1})$	0.78 (3.68)	0.81 (3.69)	0.64 (1.87)	0.46 (0.82)
$\log(\text{ISHARE}_{t-1})$	0.27 (5.83)	0.30 (5.38)	0.33 (4.97)	0.43 (3.56)
D8384_{t-1}	0.22 (3.01)	0.23 (3.07)	0.31 (3.03)	0.44 (2.51)
Short-Run Parameters				
$\Delta \log(\text{TOT}_t)$	-0.38 (-2.86)	-0.43 (-2.97)	-0.37 (-1.78)	-0.33 (-1.44)
ΔRESGDP_t	1.47 (3.29)	1.86 (3.72)	0.95 (1.27)	0.76 (0.90)
$\Delta \log(\text{OPEN}_t)$	0.38 (1.99)	0.49 (2.59)	0.29 (0.95)	0.28 (0.87)
$\Delta \log(\text{ISHARE}_t)$	0.10 (1.72)	0.10 (1.40)	0.18 (2.37)	0.11 (0.96)
$\Delta \log(\text{PFOR}_t)$	0.30 (2.39)	0.14 (1.06)	0.21 (0.97)	0.14 (0.58)
ΔD8384	0.05 (1.04)	0.05 (1.01)	0.07 (0.97)	0.04 (0.43)
Adjusted R^2	0.49	0.55	0.42	0.36
Q	14.32 (0.30)	7.21 (0.31)	7.16 (0.59)	4.68 (0.05)
DW	1.11	1.14	2.22	2.15

Note: The numbers in parentheses are *t*-ratios. The period of estimation is 1965–93. In columns 3 and 4, the long-run parameters and associated standard errors are obtained by estimating the Bewley transform of the ECM. In columns 1 and 2, we use the lagged residual from the static regression as the error-correction term. Columns 2 and 4 are instrumental variable estimates, using two lags of all right-side variables as instruments for ISHARE. The dependent variable is $\log(\text{REER})$.

Source: Computed from data from sources listed in appendix B.

correspond to the two-step Engle-Granger method and an unrestricted ECM. For Burkina Faso, where the sample is shorter and long-run coefficients are estimated imprecisely in the unrestricted ECM, we focus mainly on a parsimonious parameterization (column 3) obtained by eliminating short-run variables with statistically insignificant coefficients from the unrestricted ECM. Except when using the Engle-Granger method, long-run parameters and associated standard errors were obtained by estimating by OLS the appropriate transform of the ECM.³¹

Long-Run Parameters and Adjustment Speed

For both countries, the estimated long-run parameters strongly corroborate the theoretical model. We begin with the estimated coefficients on the resource balance to GDP ratio (RESGDP), which are positive as expected for both countries, suggesting that an increase in net capital inflows (inducing a decrease in the resource balance) raises domestic absorption and shifts the composition of potential output toward nontraded goods. The implied elasticities of the real exchange rate with respect to the resource balance (0.26 for Côte d'Ivoire and 1.02 for Burkina Faso) are comparable in magnitude to those obtained in Elbadawi and Soto (1995) for Côte d'Ivoire and Mali.

The effects of shocks to the terms of trade (TOT), as pointed out in the first main section of this chapter (on modeling the equilibrium exchange rate), are theoretically ambiguous. However, consistent with the bulk of the empirical literature (for example, Edwards (1989), Elbadawi and Soto 1995), we find that an improvement in the terms of trade appreciates the real exchange rate, suggesting that the spending effects of this variable dominate substitution effects. The estimated elasticities are plausible in light of the existing literature. Perhaps most strikingly, the magnitude of the estimated effect is very similar in the two countries despite their differences in economic structure. A 10 percent improvement in the terms of trade appreciates the real exchange rate by 4 percent in Côte d'Ivoire and 3 percent in Burkina Faso.

In both countries the estimated coefficient on the openness variable is positive, supporting the notion that trade-liberalizing reforms depreciate the equilibrium real exchange rate. The size of the elasticity differs,

31. For example, we obtain the long-run parameter estimates and their standard errors by applying instrumental variables to the Bewley transform of the ADL representation, using the ADL variables as instruments. This gives numerically equivalent results to applying OLS to the ADL, but with the advantage that the long-run parameters and associated standard errors can be read directly from the estimated equation. See Banerjee and others (1993), pp. 55–64.

Table 10.5 ECM Parameter Estimates for Burkina Faso

	1	2	3
Constant	8.94 (2.96)	7.01 (2.80)	6.12 (3.43)
Adjustment Speed			
$\log(\text{REER}_{t-1})$	-0.94 (-2.83)	-0.70 (-2.77)	-0.76 (-3.89)
Long-Run Parameters			
$\log(\text{TOT}_{t-1})$	-0.10 (-0.29)	-0.50 (-2.02)	-0.30 (-2.27)
$\log(\text{OPEN}_{t-1})$	-0.17 (-0.45)	0.18 (0.62)	0.22 (1.13)
RESGDP_{t-1}	2.28 (1.34)	4.06 (3.53)	3.89 (4.48)
$\log(\text{HBS3}_{t-1})$ (-3.22)	-1.27 (-2.84)	-0.96 (-3.48)	-0.72
$\log(\text{PFOR}_{t-1})$	0.14 (0.99)	n.a.	n.a.
Short-Run Parameters			
$\Delta \log(\text{TOT}_t)$	0.28 (0.97)	0.03 (1.16)	n.a.
$\Delta \log(\text{OPEN}_t)$	-0.13 (-0.53)	-0.02 (-0.10)	n.a.
ΔRESGDP_t	1.87 (2.01)	2.46 (3.20)	2.73 (4.34)
$\Delta \log(\text{HBS3}_t)$	-1.19 (-2.09)	-0.68 (-1.36)	n.a.
$\Delta \log(\text{PFOR}_t)$	0.07 (0.36)	0.14 (0.74)	n.a.
Adjusted R ²	0.78	0.77	0.75
Q	7.60 (0.18)	9.02 (0.11)	7.29 (0.21)
DW	2.34	2.83	1.99

Note: Numbers in parentheses are *t*-ratios. The period of estimation is 1970–93. The first column (unrestricted ECM) corresponds to equation 10.11 in the text. The long-run parameters and associated standard errors are obtained by estimating the Bewley transform of the ECM. The dependent variable is $\Delta \log(\text{REER})$.

Source: Computed from data from sources listed in appendix B.

however: it is 0.78 for Côte d'Ivoire and 0.22 for Burkina Faso. While these elasticities are not precisely estimated, they are consistent with evidence obtained by M'Bet and Madeleine (1994) and Elbadawi and Soto (1995), suggesting that the effects are stronger in the larger CFA countries.

For Côte d'Ivoire, a 10 percent increase in the share of investment in GDP (ISHARE) depreciates the real exchange rate by at least 2.7 percent, consistent with the view that this shifts the composition of spending toward traded goods. This evidence is consistent with that of Edwards (1989), but reveals an effect substantially lower than his estimates, which are in the range of 7 percent for a group of 12 developing countries. For Burkina Faso, the negative coefficient on HBS3 is consistent with a Harrod-Balassa-Samuelson effect: a 10 percent increase in domestic labor productivity compared to OECD labor productivity appreciates the real exchange rate by 7.2 percent.

To test the long-run homogeneity property—that the foreign price level, converted to CFA francs using the nominal exchange rate, does not affect the equilibrium real exchange rate—we include the log of PFOR in the specification and test the null hypothesis of a zero long-run coefficient. We use the dynamic regression results for this test since the *t*-statistics from the static regression have nonstandard distributions even under weak exogeneity. For Burkina Faso, homogeneity cannot be rejected at any reasonable level of significance (table 10.5). For Côte d'Ivoire, inclusion of the change in PFOR (or just the change in the trade-weighted nominal exchange rate) in the ECM causes a marked deterioration in the results. Thus, while long-run homogeneity cannot be rejected, the remaining results are unsatisfactory.³² For the purposes of subsequent calculations, we impose long-run homogeneity for both countries by restricting the long-run parameter on PFOR to be zero.

Short-Run Dynamics

Tables 10.4 and 10.5 show the short-run parameters from the estimated ECMs for Côte d'Ivoire and Burkina Faso. For Côte d'Ivoire (table 10.4) we show two alternatives, corresponding to the second step of the Engle-Granger procedure and the unrestricted ECM. Column 1 uses the lagged residual from the static regression in column 1 of table 10.3, so that the short-run parameters are estimated conditional on the cointegration

32. When $\Delta \log(\text{PFOR})$ is included in the regression, it soaks up much of the explanatory power of other variables. The remaining coefficients, including the long-run coefficient on LPFOR, are estimated imprecisely and often with the "wrong" signs.

vector from the static regression.³³ In column 2 we estimate the short-run parameters jointly with the long-run parameters using the unrestricted ECM.

The dynamic estimates provide direct evidence on the short-run effects of nominal devaluations on the real exchange rate. We emphasized earlier in this chapter that even under long-run homogeneity, nominal devaluations may play an important macroeconomic role if nominal rigidities prevent the price of nontraded goods from responding quickly to shocks that alter the equilibrium real exchange rate. This role requires that movements in the nominal exchange rate not be fully offset in the short run by domestic inflation. Our estimates are consistent with a transitional role for the nominal exchange rate if the coefficient on $\Delta \log(\text{PFOR})$ in the error-correction representation is positive (note also that since we are using an external real exchange rate, the upper boundary for this coefficient is not 1 but the share of traded goods in the domestic price index). For Côte d'Ivoire, the point estimates in columns 1 and 3 of table 10.4 suggest that over 20 percent of a nominal devaluation passes through to the real exchange rate over a one-year horizon. The elasticity is highest (at 0.3) in the Engle-Granger ECM (column 1), in which it is also statistically significant. An elasticity of 0.3 implies that a 50 percent nominal devaluation (as implemented in 1994: note that PFOR rises by 100 percent) will depreciate the real exchange rate by 30 percent in the short run. For Burkina Faso, the point estimates are uniformly smaller and have large standard errors. Wage-price rigidity therefore appears to give some role to the nominal exchange rate in macroeconomic adjustment in Côte d'Ivoire, but there is little evidence here of such a role for Burkina Faso.

Turning to the fundamentals, for Côte d'Ivoire we find short-run effects that are generally appreciable in size, statistically significant, and in the same direction as the long-run effects. For Burkina Faso, the short-run impact effects are substantially less than the size of their corresponding long-run coefficients, and in most cases are statistically insignificant.

A crucial parameter in the estimation of these short-run dynamic models is the coefficient of the error-correction term, which measures the speed of adjustment of the real exchange rate to its equilibrium level. The adjustment speeds estimated for Côte d'Ivoire in table 10.4 are lower (at -0.30 and -0.45, respectively, in the two-step and unrestricted ECM) than the corresponding estimate for Burkina Faso in table 10.5 (at -0.76).

33. Note that this is not the same as the error-correction representation referred to in the Granger Representation Theorem (Engle and Granger, 1987). The latter is a reduced-form equation that omits contemporaneous changes of the fundamentals.

The adjustment speed for Côte d'Ivoire is somewhat higher than that obtained for Côte d'Ivoire by Elbadawi and Soto (1995) using a similar framework. From these estimates the number of years required to eliminate a given misalignment can be derived.³⁴ For example, eliminating 95 percent of a shock to the real exchange rate would take slightly more than three years in Burkina Faso and could take as long as eight years in Côte d'Ivoire. Elbadawi and Soto (1995) find a similar difference in adjustment speed for Côte d'Ivoire and Mali. In this respect the smaller economies in the zone appear to be more adaptive to shocks than the larger ones. This conclusion is consistent with the widely held view that the latter group experienced a much higher degree of overvaluation during the 1986–94 period than the former. The results just discussed for PFOR suggest one reason for this: adjustment may be slower in these countries because nominal rigidities are more important. Slower adjustment of wages and nontraded-goods prices is consistent with a larger formal sector in Côte d'Ivoire than in Burkina Faso (here we would include both government and medium- to large-scale private enterprises) and also, for any given degree of nominal rigidity, with a larger share of nontraded goods in domestic prices. By the same token, while adjustment in Burkina Faso is relatively rapid, convergence to the new equilibrium is not immediate, suggesting the existence of some source of real rigidity of the type alluded to in the subsection on nominal rigidities and short-run dynamics.

Adjustment speeds for both countries, however, are substantially larger than the -0.19 figure obtained by Edwards (1989) using a partial adjustment model for a group of 12 developing countries with predetermined nominal exchange rates. To the degree that these adjustment speeds can be legitimately compared, they provide some support for the view that membership in a monetary union increases the credibility of monetary policy, thereby producing greater flexibility of nominal wage settlements in the private sector (Rodrik 1993).

Finally, a note on weak exogeneity for the case of Côte d'Ivoire. As discussed earlier, weak exogeneity holds with respect to the long-run parameters if the cointegrating vector does not enter the marginal model for the fundamentals. Engle and Granger (1987) suggest testing for weak exogeneity by introducing the error-correction term (the lagged residual from the static regression) into the equations of the marginal model and applying asymptotic t -tests to the hypothesis that the coefficients are zero. Using this test we are not able to reject weak exogeneity of the

34. The time required to dissipate x percent of a shock is determined according to $(1 - |\alpha|)^t = 1 - x$, where t is the number of years and α the speed of adjustment parameter.

variables individually at reasonable significance levels, with the exception of ISHARE in which we reject weak exogeneity at the 5 percent level. Rejection for ISHARE suggests problems with inference in the error-correction specification: the long-run parameter estimates remain super-consistent, but standard errors are biased and inconsistent. To handle this we re-estimate the ECM via instrumental variables, using two lagged differences of all fundamentals as instruments for ISHARE (see columns 2 and 4 of table 10.4). Inference can proceed from the IV version of the ECM, conditional on legitimacy of the chosen instruments.³⁵ The results of the IV estimation do not alter the conclusions reported above.

Step Three: Calculating the Equilibrium Real Exchange Rate

In the subsection on the relationship of the single-equation approach to the PPP approach we distinguished conditional forecasts and counterfactual simulations as two alternative approaches to constructing sustainable values for the fundamentals. Here we broaden the first of these alternatives to consider various alternatives based on the time-series behavior of the data. For policy purposes, concern often centers about the current or prospective situation rather than the historical episodes that make up the data sample. While our discussion focuses on within-sample estimates or simulations, the considerations outlined below apply equally to the construction of projected sustainable values for the fundamentals.

Sustainable Fundamentals: Time-Series-Based Estimates

When the fundamentals are stationary, their movements are inherently temporary and the conditional long-run forecast is simply the sample mean (as corrected for any deterministic trend). At the other extreme all movements in the fundamentals are permanent. In this case, the fundamentals are individually random walks and the equilibrium real exchange rate in period τ is simply βF_{τ} .

In practice, the fundamentals are likely to include both transitory and permanent components. This is clear for nonstationary fundamentals, in which the permanent component corresponds to the underlying stochastic trend.

35. Although these results are encouraging, weak exogeneity may be a more serious problem than is indicated by our variable-by-variable tests. Using Johansen's system-based chi-squared test, we strongly reject joint weak exogeneity for the fundamentals taken together.

The Beveridge-Nelson (B-N) method, which we use below in the Côte d'Ivoire case, assumes that the fundamentals each follow a univariate ARIMA($p, 1, q$) process, with the autoregressive and moving average parts generating stationary fluctuations about an underlying random walk (Beveridge and Nelson 1981). Movements generated by the unit-root part are permanent and are extracted to construct F_t^P , the permanent component of F_t . The equilibrium rate is then given by $\hat{\beta}'F_t^P$, where $\hat{\beta}$ is the vector of estimated long-run parameters. This will tend to be a somewhat smoother series than $\hat{\beta}'F_t$, reflecting the elimination of transitory shocks to the fundamentals.³⁶

We will also calculate sustainable values using centered moving averages of the fundamentals in both the stationary and nonstationary cases. This approach can be defended by appealing to the judgmental nature of the decomposition exercise and noting the disadvantages imposed by small samples. Moving averages mechanically smooth the data, to a greater degree the larger the number of periods used. In the nonstationary case, even a narrow moving average typically smooths the individual series more substantially than a B-N decomposition and may therefore yield results that are more appealing economically. The B-N approach is particularly problematic in small samples, where the results can be highly sensitive to the underlying ARIMA specification and can often exacerbate turning points in economically implausible ways. This problem can affect the resulting equilibrium rate even more dramatically: if the fundamentals are all smoothed with a moving average, the resulting equilibrium rate is simply the corresponding moving average of $\hat{\beta}'F_t$. The weighted sum of permanent components, in contrast, can easily be substantially more variable than F_t itself (as in our Côte d'Ivoire example below). Small samples also increase the possibility that stationary but persistent series are misidentified as nonstationary, in which case the B-N decomposition presumes a permanent component that in fact is not present.

In the stationary case, the moving average approach provides a way of acknowledging that even stationary fundamentals may have long-lasting movements. When a stationary variable is highly persistent, its conditional expectation at policy-relevant horizons can easily be relatively far from its unconditional mean. Using a moving average allows the long-run equilibrium rate to move in response to the current values

36. Any set of cointegrated variables has a common trend representation; this could be the basis of a joint decomposition of the real exchange rate and fundamentals into a stochastic trend component and a stationary (moving average) component (see Banerjee and others 1993). The B-N approach approximates this by treating the variables one by one.

of the fundamentals, even though these movements are thought ultimately to be temporary.

Sustainable Fundamentals: Counterfactual Estimates

Ex ante modeling of the permanent components of the fundamentals provides an important alternative to ex post approaches that rely on the underlying data-generating processes of the fundamentals. There are both positive and normative reasons for pursuing this extension. On the positive side, small samples can make it virtually impossible, when using time-series decomposition methods or moving averages, to distinguish persistent but unsustainable changes in the fundamentals from genuinely sustainable changes. The accumulation of international arrears by Côte d'Ivoire starting in the early 1980s provides an example: by this indicator, trade balances in that country appear to have been unsustainably large for over a decade. A natural approach in such a case is to construct a counterfactual path for the fundamental(s) in question that is more in line with a plausible notion of sustainability. For example, one might construct a path for the trade balance that would have kept arrears reasonably low given "voluntary" capital inflows. The exercise will often require a sequence of judgments; in this case, one needs a plausible description of voluntary inflows, and one may be as interested in the sensitivity of the estimated misalignment to changes in assumptions as in the overall change relative to the baseline.

The second, more normative use for counterfactual simulations is in addressing the "what if" questions that are of central interest to policymakers, particularly when the fundamentals include variables potentially under policy control. Again using the Côte d'Ivoire case, policymakers might want to know the implications for the real exchange rate of a trade liberalization or change in government spending patterns. Preserving the relative simplicity of the single-equation approach, a natural way of handling these concerns is to construct counterfactual simulations of desirable values for selected fundamentals. As in the positive case, the construction of "desirable" values for the fundamentals is not a trivial exercise. Theory can often provide loose guidelines (for example, in the proposition that the optimal tariff is zero for a small open economy with no other distortions), but translating these into alternative values for the fundamentals will require an additional set of judgments (in this case, assessing what freer trade would have meant for the openness ratio, which is our proxy for trade policy).

As pointed out in appendix C, a potentially important side effect of counterfactual simulations, whether the underlying motivation is positive or normative, is to break the restriction implicit in the methodology that the average degree of misalignment be near zero within sample.

The reason is straightforward: the misalignment calculation is now done using time paths for the fundamentals that were not used estimating the long-run parameters. The implicit super-exogeneity assumption, as emphasized earlier, is that the β vector estimated using sample information is relevant for assessing the effect of alternative paths for the fundamentals.

In appendix C, we construct counterfactual simulations for the resource balance, openness, and investment share variables for both Côte d'Ivoire and Burkina Faso. For Côte d'Ivoire, the simulations incorporate the following judgments (using "unsustainable" and "undesirable" to distinguish essentially positive rationales from essentially normative ones):

- The actual resource balance was unsustainably low after 1979;
- Trade policy was undesirably restrictive, particularly after 1979; and
- The investment to GDP ratio was undesirably low, particularly after 1979.

For Burkina Faso, in which the investment to GDP ratio does not enter the model, the key judgments are:

- The resource balance is determined by the volume of concessional inflows, and drought-year levels are unsustainable; and
- Trade policy was undesirably restrictive throughout the sample.

The details of these calculations appear in appendix C.

Estimating the Degree of Misalignment

The estimated degree of misalignment, m_t , is simply the percentage difference between the real exchange rate and its computed equilibrium value, as expressed in equation 10.15:

$$(10.15) \quad m_t = \ln e_t - \ln e_t^* = [\ln e_t - \hat{\beta}' F_t] + \hat{\beta}' (F_t - F_t^p).$$

For within-sample estimates, e_t is simply the actual real exchange rate. For out-of-sample estimates, e_t can be forecasted using a dynamic simulation that feeds projected paths for the fundamentals through the estimated short-run parameters of the model.

The degree of misalignment is decomposed mechanically in equation 10.15 into an error-correction term that captures the deviation of the exchange rate from the "fitted" real exchange rate using long-run parameters (the term in square brackets) and a term that captures the

deviation of the current fundamentals from sustainable values. Expressing m_t this way brings out the role of sustainability calculations for the fundamentals. Suppose, for example, that the long-run parameter for the terms of trade is negative, implying that a sustained terms-of-trade improvement appreciates the real exchange rate. If most movements in the terms of trade are temporary, however, and households optimize without borrowing constraints, then the short-run impact of a change in the terms of trade should be substantially below the estimated long-run impact (as in our theoretical model). A temporary improvement in the terms of trade would then produce offsetting changes in the components of m_t . The second component would be large and negative, reflecting the temporary nature of the terms of trade boom; the first would be large and positive, reflecting the very modest response of the actual real exchange rate to the substantial short-run movement in F_t . Misalignment calculated using the actual rather than sustainable value of the terms of trade (that is, setting $F_t^p = F_t$) would pick up only the second of these effects, producing the mistaken impression of a badly undervalued real exchange rate.

What the decomposition cannot do, of course, is identify the source of misalignment relative to plausible values for F^p . As discussed earlier, e_t may differ from F_t^p for reasons of real or nominal rigidities or, equivalently, equilibrium or disequilibrium dynamics; or it may be pushed by random shocks.

Empirical Results: Equilibrium Real Exchange Rates and Misalignment

Tables 10.6 and 10.7 show alternative measures of the equilibrium real exchange rate while figures 10.5 and 10.6 depict the observed and sustainable RERs as well as the fitted (for Côte d'Ivoire) and trend (for Burkina Faso) real exchange rates.³⁷ For Côte d'Ivoire, we use the long-run parameters derived from the static regression in column 1 of table 10.3. For Burkina Faso, we use the long-run parameters from the unrestricted ECM in column 1 of table 10.5.

We report four measures of the equilibrium real exchange rate for Côte d'Ivoire: the fitted RER, its corresponding five-year moving

37. Figures 10.5 and 10.6 could have been supplemented by confidence intervals based either on the standard errors of the estimated parameters or (via bootstrapping) on the empirical distribution of the data. Bootstrapping confidence intervals, however, are in general quite wide and, given the imprecision of our parameter estimates, are likely to be so in our case.

Figure 10.5 *The RER for Côte d'Ivoire, 1965 to 1993 (1985 Actual RER=100)*

Note: An upward movement is an appreciation of the RER.

Figure 10.6 *The RER for Burkina Faso, 1970 to 1993 (1985 Actual RER=100)*

Note: An upward movement is an appreciation of the RER.

average, an equilibrium rate based on Beveridge-Nelson decompositions of the fundamentals, and one based on the counterfactual simulations described in appendix C. For Burkina Faso, we replace the B-N decomposition with the fitted trend for the real exchange rate; as

discussed earlier, this represents the most natural long-run forecast for a trend-stationary variable.

Recall that when we generate long-run "forecasts" of the real exchange rate using time-series-based estimates of the "permanent" fundamentals, we require not only adequate estimates of the long-run parameters but also a lack of Granger causality from the real exchange rate to the fundamentals. With a lag length of one, weak and strong exogeneity coincide and the partial tests reported earlier for Côte d'Ivoire therefore provide some support for these calculations. As an additional check, we tested the multivariate generalization of Granger noncausality from the real exchange rate to the fundamentals and were unable to reject noncausality at any reasonable levels, using a lag length of either one or two. As argued earlier, the use of counterfactual simulations for the fundamentals involves an assumption that the long-run parameters are invariant to the interventions being constructed; we treat this as a maintained hypothesis.

The last columns of tables 10.6 and 10.7 show the percentage gap between the observed and equilibrium real exchange rates, using the counterfactual simulations for the equilibrium rate. The gap between these two series provides a measure of real exchange rate misalignment. The figures show a remarkable success on the part of the computed index in reproducing well-known overvaluation (and undervaluation) episodes in the recent macroeconomic history of these countries and the CFA zone more generally. In particular, note that Côte d'Ivoire managed to reverse substantial real overvaluation by 1985–86. While some of this was generated by contractionary macroeconomic policies that fell heavily on investment, a substantial contribution came from the steady depreciation of the French franc against the U.S. dollar and other major currencies and an ultimately temporary recovery in the terms of trade. When the French franc moved in the reverse direction following 1986, the fiscal laxity and structural rigidities that characterized the Côte d'Ivoire economy all along were fully exposed; our calculations imply that during the 1987–93 period the real exchange rate was overvalued by 34 percent on average. By 1994 a set of corrective measures, including a zone-wide 50 percent devaluation, were implemented. Using a constant elasticity model, Devarajan (Chapter 8) finds a somewhat larger degree of overvaluation in Côte d'Ivoire for 1993 (56 percent in domestic-currency terms) than our estimates based on counterfactual simulations of the fundamentals.

For Burkina Faso, in contrast, our results for 1980–93 do not indicate any major overvaluation (last column of table 10.7 and figure 10.6). Indeed, according to our estimates, Burkina Faso's real exchange rate was undervalued by 1 percent on average between 1980 and 1986 and by

Table 10.6 Observed and Equilibrium RER Indexes for Côte d'Ivoire—1980 to 1993 (1985 Observed RER=100)

Year	Equilibrium RER					
	Observed	Fitted	5-year MA	B-N	"Sustainable"	Overvaluation
1980	72	77	85	74	108	51
1981	83	80	93	81	107	29
1982	91	85	94	86	101	10
1983	96	102	95	83	94	-3
1984	100	95	87	82	76	-24
1985	100	90	76	96	90	-10
1986	79	76	82	87	84	6
1987	67	71	85	83	98	46
1988	67	68	80	96	103	54
1989	70	67	75	54	92	32
1990	66	64	79	52	83	26
1991	66	68	76	60	91	38
1992	61	71	81	60	93	52
1993	60	65	80	64	85	41

Note: The observed RER is the one used in the econometric analysis. The long-run parameter vector is taken from the static regression in column 1 of table 10.3. "Fitted" values are obtained directly from that regression; "5-year MA" refers to five-year moving averages for all fundamentals; "B-N" refers to Beveridge-Nelson decompositions of all fundamentals; and the "sustainable" RER is defined as the fitted RER with all fundamentals replaced by counterfactual sustainable values, as determined in appendix C. Overvaluation is defined as $100 \cdot (\text{sustainable RER} - \text{observed RER}) / (\text{observed RER})$.

Source: Computed from data from sources listed in appendix B.

nearly 14 percent during 1987–93.³⁸ The estimated undervaluation may be on the high side for the latter period.³⁹ Burkina Faso is generally regarded, however, as having adjusted more successfully to the adverse shocks that affected the entire zone after 1986, especially in comparison with the larger CFA countries (Devarajan and Hinkle 1995). Substantially milder overvaluation (or even undervaluation) is one measure of this success; another is the absence in Burkina Faso of the deep reces-

38. The apparent overvaluation in 1980 was eliminated by the depreciation of the actual REER in the early 1980s that was caused largely by the depreciation of the French franc to which the CFA franc was linked.

39. For example Elbadawi and Soto (1995), using a similar methodology, estimate that the RER in Mali was virtually in equilibrium (on average) during the 1987–94 period, while the CGE estimates of Devarajan in Chapter 8 suggest that the RER in Burkina Faso was overvalued by about 9 percent in 1993.

Table 10.7 Observed and Equilibrium RER Indexes for Burkina Faso—1980 to 1993 (1985 Observed RER=100)

Year	Equilibrium RER					
	Observed	Fitted	Trend	5-year MA	"Sustainable"	Overvaluation
1980	86	100	94	101	104	17
1981	94	100	95	101	102	7
1982	93	103	95	104	98	6
1983	97	103	96	103	86	-13
1984	104	113	96	102	79	-31
1985	100	98	96	101	87	-16
1986	95	93	97	100	91	-4
1987	97	100	97	98	96	-1
1988	98	99	97	100	96	-2
1989	102	102	98	101	93	-10
1990	99	106	98	100	94	-6
1991	98	99	98	101	95	-4
1992	98	98	99	100	98	0
1993	96	99	99	99	100	-4

Note: The observed RER is the one used in the econometric analysis. The fitted RER is the one estimated from the cointegration regression (see table 10.6). "Trend" refers to a fitted linear trend for the RER. "5-year MA" refers to five-year moving averages. The "sustainable" RER is the fitted RER in which the fundamentals (RESGDP and OPEN) have been replaced by their sustainable counterparts as outlined in appendix C. Overvaluation is defined as $100 \cdot (\text{sustainable RER} - \text{observed RER}) / (\text{observed RER})$.

Source: Computed from data from sources listed in appendix B.

sion experienced by Côte d'Ivoire during the 1980s and early 1990s. Both observations suggest a more flexible domestic wage and price structure in the smaller of the two countries, and therefore significantly milder nominal rigidities.

Conclusions

The decision to devalue depends fundamentally on the degree of misalignment of the real exchange rate and the speed with which internal adjustment mechanisms are likely to restore macroeconomic balance. Measuring the degree of misalignment is difficult, however, given that the equilibrium real exchange rate is unobservable whenever the economy is not in internal and external balance. The standard PPP approach is to identify a period in which the economy is judged to have been in balance, and to take the real exchange rate of that period as the equilibrium rate for all years. But this fails to account for the effect of changes in the fundamentals on the equilibrium real exchange rate.

Once the endogeneity of the equilibrium real exchange rate is recognized, however, a second problem arises: restricting attention to plausible candidates for years of macroeconomic balance, there will rarely be enough observations to estimate the elasticities of the equilibrium rate with respect to even a small list of fundamentals. In this chapter, we addressed these problems by imposing the relatively mild (and testable) restriction, drawn from standard theories of the equilibrium real exchange rate, that the distance between the actual and equilibrium real exchange rates is a stationary random variable. When the variables are $I(1)$, this leads naturally to the use of cointegration methods for estimating the long-run relationship between the real exchange rate and its fundamentals. When the variables are stationary, standard procedures of dynamic specification and estimation apply. We illustrated the methodology using annual data for Côte d'Ivoire and Burkina Faso.

How useful an addition is this methodology to the standard toolbox for assessing the equilibrium real exchange rate and the degree of misalignment? Our view is that this methodology belongs in the analyst's toolkit as a clear advance over PPP and a useful complement to other methods. There are three fundamental reasons for this.

- First and foremost, this approach provides a natural way of incorporating the reality that the fundamentals will sometimes move permanently. In such a case our approach extracts maximal leverage from the theoretical proposition that the real exchange rate will not stray indefinitely from a function of the fundamentals.
- Second, estimating the equilibrium real exchange rate is typically motivated by policy concerns. The analyst may therefore be particularly interested in the relationship between the equilibrium real exchange rate and hypothetical changes in individual fundamentals. For out-of-sample exercises, interest would center on how changes in the fundamentals would alter both the actual and the equilibrium rates, and thereby the degree of misalignment. Under super-exogeneity of the fundamentals, our approach delivers a set of parameters that can be used for such policy analysis in a transparent and straightforward manner.
- Third, this approach takes a partial step toward imbedding the determination of the long-run relationship in the overall dynamic relationship between the real exchange rate and its fundamentals. Under weak exogeneity with respect to the short-run parameters, fully efficient estimation and inference on these parameters can take place conditional on the current and lagged fundamentals. The resulting information on short-run dynamics may be of interest in its own right and if Granger noncausality also holds, can be

used to generate short-term forecasts of the real exchange rate and degree of misalignment.

From the viewpoint of dynamic specification, there are various directions in which the approach advocated here might be extended. One is to allow both $I(0)$ and $I(1)$ variables in the long-run relationship. In this case, the theory still implies cointegration among the $I(1)$ variables, but the Engle-Granger two-step method will produce inconsistent estimates of the long-run parameters on the stationary variables. We are therefore pushed toward allowing explicitly for the short-run dynamics, whether via the error-correction model, the Johansen procedure, or some alternative. A second extension would be allowing multiple long-run relationships between the variables. Such a case might arise, for example, from the existence of a reaction function relating fiscal policy to the trade balance or the real exchange rate. Moreover, since most of our variables are already measured in ratios (the real exchange rate, the openness variable, and so on), we may already be reducing the order of integration of underlying nonstationary variables (such as the domestic price level). The structural error-correction model of Boswijk (1995) represents the closest counterpart to our analysis in the case of multiple cointegrating relationships. Finally, we have chosen not to impose any theoretical structure on the short-run dynamics. In cases where particular sources of short-run dynamics are of interest, there may be a substantial return to developing a theoretical structure to capture these dynamics, and imposing the resulting identifying restrictions; for an interesting attempt to incorporate rigidity of domestic prices, see Kaminsky (1987). An important challenge along these lines is that of separating misalignments caused by price rigidities from those caused by internal real dynamics or temporary movements in the fundamentals. Naturally, most of these extensions will bring out a tension between maintaining the simplicity of a single-equation approach—an important feature if this approach is to be used “in the field”—and allowing the overall dynamic relationship(s) to emerge from the data.

As a final extension of the single-equation approach, we note the possible usefulness of cross-country data in tying down the long-run elasticities. A version of our static regression could easily be run on a pure cross-section or panel of countries. The obvious advantage of this approach lies in its expansion of the sample size. The resulting increase in degrees of freedom is conditional on the validity of pooling restrictions, but with multiple time periods some of these will be testable. The handling of dynamics remains a difficult problem in panel data, however, and in this area there is a clear tradeoff between the flexibility associated with our single-country approach and the restrictions required to

support dynamic estimation in a panel. Our theoretical model and empirical results suggest that pooling restrictions are at least as likely to fail with respect to short-run parameters and error structure as with respect to the long-run parameters. Strategies such as using nonoverlapping time-averaged data (for example, five-year averages) may help minimize some of these difficulties but to our knowledge a consensus has not yet emerged on how to handle nonstationarity in panel data.

In the field, of course, the virtue of cross-sectional or panel results is that the long-run parameters can be "borrowed" from existing studies without requiring new estimation. Such parameters could be combined with data on changes in a given country's fundamentals to derive changes in the equilibrium real exchange rate for that country and therefore changes in the degree of misalignment. Identifying the *level* of misalignment in any particular year would then require a "rebasings" exercise of some sort, and in this respect the use of borrowed cross-country parameters is a kind of hybrid of the PPP approach and econometric approaches.

Our aim in this chapter has been to give a self-contained presentation of a methodology that we consider to be applicable at reasonably low cost in the field. The chapter is not a cookbook, however. In the end, the effective use of the single-equation, time-series approach requires a balanced sense of both its virtues and its limitations and—as always in econometric practice—some attention to the evolving state of time-series econometrics. We close by identifying three particular cautions regarding the use of our methodology in the policy arena.

First, the econometric approach is data intensive and inherits all the limitations of developing-country data. Our empirical findings for Côte d'Ivoire and Burkina Faso are broadly consistent with the empirical literature on equilibrium real exchange rates in developing countries, and they line up well with estimates obtained by other methods. But they are not robust. We noted above that the econometric results were quite sensitive to the choice of proxies for the fundamentals and to the estimation procedure. The choice of real exchange rate index also made a difference empirically, and although changes in long-run elasticities are to be expected, we found that the overall statistical performance was highly sensitive to whether an internal or external real exchange rate concept was employed and whether the nominal exchange rate was adjusted for black market transactions. While such conditions define the art of econometric practice, they may be particularly acute when the notion of long-run equilibrium is required to carry so much weight in short samples.

The reality of short samples brings out a second potential weakness in this approach, even relative to the PPP approach. In effect, the single-equation methodology assumes that the economy was in internal and

external balance *on average* over the sample period. This avoids the need for a priori and possibly ad hoc claims about macroeconomic balance in any particular year, providing instead a systematic way of bringing the whole sample to bear in determining the path of the equilibrium rate. But it implies that unless the analyst is prepared to undertake counterfactual simulations for the fundamentals, the average degree of misalignment in the sample will tend, by construction, to be small. There will be little scope for uncovering episodes of overvaluation or undervaluation that last more than a few years. In the CFA zone, where the real exchange rate was widely thought to be overvalued for most of the period between 1978 and 1994, the implicit "balance on average" assumption may be seriously misleading. The PPP approach, of course, does not require such an assumption; the result is that the real exchange rate can in principle be overvalued (or undervalued) in every period other than the benchmark one. We suggested that a natural way of handling this within our methodology was to construct counterfactual simulations for the fundamentals. In our counterfactuals for Côte d'Ivoire, freer trade, higher domestic investment, and smaller trade deficits all produced a depreciation of the equilibrium rate and therefore tended to increase the estimated degree of misalignment.

Finally, the methodology relies on concepts of equilibrium and misalignment that are conditional on policies or structural features that can reasonably be treated as predetermined, whether or not those policies or features generate welfare losses. In this view, short-run misalignments may well reflect market-clearing responses to shocks, and long-run movements in the real exchange rate may well reflect highly suboptimal macroeconomic policy choices. For both reasons the misalignments most readily identified using single-equation time-series methods—those not requiring counterfactual simulations—may not be the most interesting from a policy perspective. While we have seen that policy content can be imposed in the form of normative counterfactual simulations for the fundamentals, the implicit assumption of super-exogeneity places an additional burden on the data that may or may not be justified in the sample at hand.

Appendix A

Conditioning and Weak Exogeneity

Weak exogeneity is (potentially) a property of the joint distribution of the real exchange rate and the fundamentals. In this appendix we introduce the concept of conditional and marginal models and explore the relationship between the single-equation model (equation 10.11) and the full distribution of the $(nx1)$ vector $x_t = [\ln e_t, F_t', s_t']'$, conditional on its own past (see also Ericsson 1992). With reasonable generality we can describe this distribution as the p^{th} -order Gaussian vector autoregression (VAR), as expressed by equation 10.A.1:

$$(10.A.1) \quad x_t = \sum_{j=1}^p \Pi_j x_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim IN(0, \Sigma)$$

where the Π_j are (nxn) matrices of reduced-form coefficients and Σ is the nxn symmetric and positive definite matrix of contemporaneous covariances between the innovations ε_{it} . Equation 10.A.1 can be written equivalently as equation 10.A.2:

$$(10.A.2) \quad \Delta x_t = \Gamma x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t$$

where $\Gamma = [(\Sigma_j = \sum_{i,j} \Pi_j) - I]$ and $A_j = -\sum_{i=j+1,p} \Pi_i$. The first row of equation 10.A.2 is a *reduced-form* error-correction model for $\Delta \ln e_t$; it is similar to equation 10.11 but excludes contemporaneous values of F and s . To obtain the distribution of $\Delta \ln e_t$ conditional on lagged x_t and contemporaneous F and s , we first partition the vector x_t into $x_t = [\ln e_t, w_t']'$, where $w_t = [F_t', z_t']'$ is the vector of macroeconomic determinants of the real exchange rate. Without loss of generality, we can then factorize the joint distribution represented by equation 10.A.2 into the distribution

of $\Delta \ln e_t$ conditional on contemporaneous w_t 's and lagged x_t 's and the associated marginal distribution of the w_t 's given lagged x_t 's. Under normality of ε_t , the conditional and marginal models take the form shown in equation 10.A.3.a:

$$(10.A.3.a) \quad \Delta \ln e_t = \Sigma_{12}(\Sigma_{22})^{-1} \Delta w_t + (\Gamma_1 - \Sigma_{12}(\Sigma_{22})^{-1} \Gamma_2 x_{t-1} + \sum_{j=1}^p (A_{1j} - \Sigma_{12}(\Sigma_{22})^{-1} A_{2j}) \Delta x_{t-j} + \xi_t$$

$$(10.A.3.b) \quad \Delta w_t = \Gamma_2 x_{t-1} + \sum_{j=1}^p A_{2j} \Delta x_{t-j} + \varepsilon_{2j}$$

where the numerical subscripts refer to the blocks of appropriately partitioned matrices (so that, for example, Γ_1 is the first row of Γ and Σ_{22} the $n-1 \times n-1$ lower-diagonal bloc of Σ). By construction, the disturbance term in (10.A.3.a), $\xi_t = \Sigma_{11} - \Sigma_{12}(\Sigma_{22})^{-1} \Sigma_{21}$ is uncorrelated with all of the variables on the right-hand side of that equation. That this representation is simply a reparameterization of (10.A.2) can be confirmed by premultiplying equation 10.A.2 by the $n \times n$ nonsingular matrix

$$B = \begin{bmatrix} 1 & \Sigma_{12}(\Sigma_{22})^{-1} \\ 0 & I_{n-1} \end{bmatrix}$$

which results in equation 10.A.3.

Equation 10.A.3.a is a single-equation conditional error-correction model whose form mimics that of equation 10.11. Although it is often assumed in writing an equation like 10.11 that the disturbance is uncorrelated with the right-hand side variables, this is true *by construction* for equation 10.15.a. To the degree that the parameterizations differ, therefore, OLS estimation of equation 10.11 will tend to uncover the parameters of equation 10.A.3.a (in which orthogonality holds by construction), yielding inconsistent estimates of the parameters of equation 10.11. Moreover, even if the parameters of equation 10.11 can be recovered from those of equation 10.A.3.a, the latter are potentially complicated functions of the underlying VAR parameters. There may therefore be cross-equation restrictions linking these parameters to those of the marginal model (equation 10.A.3.b). In such a case efficient estimation of the conditional model requires that these restrictions be imposed; and failure to impose them may produce inconsistent standard errors, invalidating inference.

These considerations motivate a search for conditions under which estimation and inference regarding particular parameters of equation

10.11 can proceed successfully in the conditional model alone (in other words, without analyzing the full system). In such cases the subvector w_t is said to be *weakly exogenous* for the parameters of interest (Engle, Hendry, and Richard 1983). In the context of the above discussion, weak exogeneity requires (a) that the parameters of interest can be directly recovered from those of the conditional model and (b) that there be no cross-equation restrictions linking these parameters to those of the marginal model.

Weak exogeneity is testable, though generally at the cost of moving to systems estimation. For the case of nonstationary but cointegrated variables (see the section on the $I(1)$ case), Urbain (1992) and Johansen (1992) show that w_t is weakly exogenous for the long-run parameters and adjustment speed if $\Gamma_2 = 0$, or equivalently if the cointegration vector does not enter the marginal model. In our empirical section we test this restriction for the case of Côte d'Ivoire. With respect to the short-run parameters of equation 10.11, matters are more complicated. The condition for weak exogeneity with respect to the long-run parameters of equation 10.11 also guarantees weak exogeneity with respect to the short-run parameters *of the conditional model itself* (that is, of equation 10.A.3.a). Recall, however, that the long-run parameters of interest were derived not from conditioning but from a theoretical model. If the short-run parameters (equation 10.11) have similar structural interpretations, then the conditions for weak exogeneity are more demanding. A set of sufficient conditions (Urbain 1992) when the variables are nonstationary and cointegrated is $\Gamma_2 = 0$ and $\theta = 0$, where θ is the vector of covariances between the disturbance in equation 10.11 and the vector of disturbances from the marginal model (equation 10.A.3.b). (Under these conditions, equations 10.A.3.a and 10.A.3.b form a block-recursive system.) We do not test for weak exogeneity of the short-run parameters in this chapter.

When the variables are stationary, the lack of a clear statistical distinction between their individual and joint variation carries over to the conditions for weak exogeneity, which now make no general distinction between the short- and long-run parameters. A sufficient condition in the limited information context of this chapter (that is, the context in which identifying restrictions on the marginal model are not available) is that equation 10.11 and the marginal model form a block-recursive system. As is well known, this guarantees predeterminedness and obviates the need for instrumental variables. We will not formally test for weak exogeneity in the $I(0)$ case (see for example, Monfort and Rabemanajara 1990), treating it instead as a maintained hypothesis where necessary.

Appendix B

Data Description

The data were taken from three sources: (a) IMF, International Financial Statistics; (b) UNCTAD; and (c) the World Bank's Unified Survey. The variables were constructed as follows:

Real Exchange Rate (RER). The ratio of the trade-weighted index of foreign wholesale prices each expressed in CFA (local currency) terms by converting at the relevant bilateral official exchange rate to the home country's consumer price index (CPI). The price and exchange rate indexes (WPI and NER) are calculated as geometric averages across the home country's n largest trading partners, with bilateral total (import plus export) trade shares (normalized to unity) as weights. We use official data for the trade weights; these are not corrected for unrecorded trade: $RER = (WPI \cdot NER)/CPI$.

Terms of Trade (TOT). The external terms of trade are P_X^w/P_M^w , where P_X^w and P_M^w are export and import price indexes (expressed in dollars) from UNCTAD. The macroeconomic impact of a change in the terms of trade is proportional to the share of international trade in economic activity. If the export share is relatively constant over the sample there is little point in adjusting the relative price measure. Our data for Côte d'Ivoire, however, show what appears to be a major structural increase in exports starting in 1984. To capture this feature we multiply Côte d'Ivoire's external terms of trade by an export share dummy variable, defined for observations between 1965 and 1983 inclusive as the average export share for that period and for later observations as the average export share after 1984.

Openness (OPEN). OPEN1 is the import to GDP ratio (IMPGDP), and is defined as the value of imports at current prices (IMPCP) over GDP at current prices (GDPCP): $OPEN1 = IMPCP / GDPCP$. OPEN2 is the ratio of the value of imports at constant prices (IMPKP) plus exports

at constant prices (EXPKP) to GDP at constant prices (GDPKP): $OPEN2 = (IMPKP + EXPKP)/GDPKP$. $OPEN3$ is the ratio of imports at constant prices to domestic absorption at constant prices: $OPEN3 = IMPKP / (GDPKP - (EXPKP - IMPKP))$.

Resource Balance to GDP Ratio (RESGDP). Value of exports at constant prices (EXPKP) minus value of imports at constant prices (IMPKP), divided by GDP at constant prices (GDPKP). EXPKP has been adjusted by the domestic terms of trade (TOTD), which are defined as the ratio of export to import deflator. Thus $RESGDP = (EXPKP \cdot TOTD - IMPKP) / GDPKP$.

Investment Share (ISHARE). Ratio of gross investment at constant prices (IGROSS) to the sum of private consumption (PCONK), government consumption (GCONK), and gross investment, all at constant prices: $ISHARE = IGROSS / (PCONK + GCONK + IGROSSK)$.

Foreign Price Level (PFOR). Trade-weighted index of foreign wholesale prices expressed in CFA terms (that is, in home-country currency). Thus $PFOR = WPI \cdot NER$ (and $RER = PFOR / CPI$; see definition of RER above).

Harrod-Balassa-Samuelson Proxy (HBS3). A lagged 3-year weighted moving average of the ratio of home country GDP per worker to OECD GDP per worker, using the Penn World Tables (Heston-Summers) data for these variables. OECD GDP per worker was constructed by summing OECD GDP and dividing by total OECD workers. Weights decline linearly. Denoting the ratio of GDPs per worker in year t by $R(t)$: $HBS3(t) = (3/6) \cdot R(t-1) + (2/6) \cdot R(t-2) + (1/6) \cdot R(t-3)$.

Appendix C

Sustainable Fundamentals

Time-Series Measures: TOT and LPFOR

Both Burkina Faso and Côte d'Ivoire are very small economies by world standards and are therefore price takers in the markets for both their exports and imports. Moreover, the nominal exchange rate for the CFA francs was fixed throughout the 1970–93 sample period and could not be changed by individual CFA countries. The terms of trade (TOT) and the foreign price level converted to CFA francs (LPFOR) are therefore exogenous variables. While these variables fluctuate substantially from year to year, we have no basis on which to question the sustainability of their longer-run movements. We therefore use five-year centered moving averages as the sustainable values of these variables (extrapolating out of sample using the first- and last-year values). We also generate alternative sustainable values for Burkina Faso and Côte d'Ivoire using sample means and Beveridge-Nelson decompositions, respectively.

Counterfactual Simulations: RESGDP

RESGDP is the ratio of the resource balance to GDP, both in constant prices. Since Burkina Faso relied heavily on concessional aid flows in 1970–93, determining a sustainable resource balance is essentially a problem of determining sustainable levels of financial inflows. These inflows can be divided into net factor income, net transfers, and net capital flows. We used five-year moving averages for the first two (interest payments were small and changed very slowly over the sample, so we ignored the feedback from borrowings to interest payments). We then divided net capital flows into its dominant component—net long-term concessional borrowing—and “other” flows (net direct investment, net portfolio investment, net short-term borrowing, net errors and omissions), using five-year moving averages for the latter. The government of Burkina

Faso attempted to maximize net concessional borrowing during the sample period, so this component was ultimately determined by the foreign donors. To smooth out year-to-year fluctuations in net concessional borrowing, we used the smaller of the five-year moving average of the actuals or 3.5 percent of GDP (the highest level reached except in drought years). The sustainable resource balance is then the sum of these sustainable components. Note that the World Bank's debt stock and flow data are not consistent with the national accounts and balance-of-payments data for Burkina Faso and Côte d'Ivoire. Since the balance of payments and national accounts data are consistent with each other and essential for the analysis, we used balance-of-payments data when there were conflicts between these and the Bank's debt data.

The Côte d'Ivoire case is both more complicated and more representative of the problems likely to emerge in developing-country applications. Côte d'Ivoire avoided balance-of-payments and debt problems in the 1970s. We therefore treated actual flows as essentially sustainable during the 1965–79 period, using five-year moving averages to smooth out temporary fluctuations. After 1980, the country was unable to meet its debt service payments. Moving averages therefore seem unlikely to capture sustainable movements in net borrowing and interest payments after 1980, and we cannot ignore the feedback from higher debt levels to higher interest payments. For 1980–93 we proceed as follows.

To proxy the sustainable level of borrowing, we used zero net repayments and net disbursements after 1979 (in other words, no change in the debt stock other than through write-downs). Côte d'Ivoire's debt ratio jumped from 47 percent in 1979 to 62 percent in 1980, then climbed to 115 percent in 1985, after which the country defaulted. The Maastricht Treaty, after which the fiscal guidelines for the West African Monetary Union are modeled, sets 60 percent of GDP as the maximum desirable debt level for the EU countries. A developing country might be able to target a somewhat higher debt level than 60 percent depending upon its rate of growth and its access to financing on concessional terms; so 1979 is by these criteria the last year of sustainable debt levels.

We calculate sustainable direct and portfolio investment as assumed percentages of total sustainable investment as determined below; together with the sustainable borrowing figures, these yield a sustainable level of total capital inflows.

To proxy sustainable interest payments, we use 4 percent of GDP. This represents a kind of compromise between a normative scenario in which interest payments are capped at 2.5 percent of GDP and a positive scenario (essentially feasibility calculation) that caps them at 5 percent. For comparison, the Maastricht debt ceiling, with an inflation rate of 3 percent and a real interest rate of 3 percent, implies interest payments of 1.8 percent of GDP for the EU countries. Côte d'Ivoire was

unable to sustain the service payments on its debt after interest payments reached 3.5 and 5.2 percent of GDP in 1981 and 1982.

The sustainable resource deficit for 1980–93 is then calculated as the sum of net transfers, net factor income, and net capital inflows, using five-year moving averages of the actuals for transfers and factor income flows other than interest payments.

Counterfactual Simulations: ISHARE and OPEN1

ISHARE is the ratio of investment to GDP in constant prices; OPEN1 is the ratio of imports to absorption in current prices. The sustainability criterion we use for these variables is consistency with a 3 percent long-run growth rate of GDP per capita.

With population growth estimated at about 3 percent for both countries over the sample, GDP growth of 6 percent is required to achieve 3 percent growth in GDP per capita. Using ICORs of 4 for Côte d'Ivoire and 5 for Burkina Faso, this would in turn require investment ratios of about 25 percent and 30 percent of GDP, respectively. The 25 percent ratio is in line with those actually achieved during the 1960s and 1970s in Côte d'Ivoire; it is also the target that the World Bank has suggested as a guideline for Africa as a whole (World Bank 1989). For Côte d'Ivoire, therefore, we use a moving average of the actual investment levels for 1965 to 1981, which were reasonably close to 25 percent, and 20 percent for 1982–93 when investment was depressed far below this level. For Burkina Faso, in which the investment to GDP ratio is used only as an input to calculate the target import to absorption ratio (see below), we assume a sustainable investment ratio of 25 percent.

For both countries, we assume that increases in the import to GDP ratio were required to deliver the import content of additional investment and also support a more liberal trade regime. We estimate an import content of investment of roughly 0.6 for both countries. To incorporate trade liberalization, we assume increases in the import ratio of 3 percent and 2 percent of GDP, respectively, for Côte d'Ivoire and Burkina Faso. The target import ratio is then estimated as the actual import ratio plus 3 percent of GDP plus 0.6 times the difference between the target investment ratio and the actual investment ratio. This target import to absorption ratio is used for the entire sample period, as a more open trade policy would have been desirable throughout.

A Caveat

As the above discussion suggests, determining target values for particular countries requires considerable country-specific knowledge and a number of assumptions based on partial information and analysis. These assumptions are open to question—and different ones (regarding

either the key parameters or the underlying notion of sustainability) would yield different results. It may therefore be important in specific cases to consider alternative plausible assumptions and to compare the results of the various alternatives to those from using moving averages for the target variables.