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Review Of “Constructive Combinatorics” By D. Stanton And D. White

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Reviews

Stanton, Dennis, and Dennis White. 1986. *Constructive Combinatorics.* New York: Springer-Verlag. x + 183 pp. $20. ISBN 0-387-96347-2.

There has been growing interest in the algorithmic approach to mathematics. For most of the last 50 years, mathematicians have preferred to prove things by existential arguments, considering constructions a side show for others. Now, however, many feel that algorithms can play a central role. If you have an algorithm to construct something, you can quickly construct enough examples to formulate conjectures. More important, the algorithm proves the existence of the objects, and an analysis of the algorithm may lead to further theorems about the nature of the objects. *Constructive Combinatorics* is an excellent example of the power and charm of this modern viewpoint.

However, it is important to say what this book is not. The back cover calls it an "introductory text." It is not. The title suggested to me that it surveys combinatorics, but it does not. What it *does* do is clearly set forth in the preface. It was written for the third quarter of a junior-senior combinatorics sequence in the Mathematics Department at the University of Minnesota, following quarters in enumeration and graph theory. The third quarter had been about doing combinatorial algorithms, but the authors felt such a course did not have enough mathematical content. So they wrote a book with an algorithmic viewpoint but which was devoted to combinatorial topics, notably topics with an algebraic flavor.

Chapter 1 is the most algorithmic chapter. Algorithms are given (in a simplified Pascal-like language) for listing various combinatorial objects and for going directly to and from the n^{th} object in the list. Chapter 2 discusses partially ordered sets and shows how some of the algorithms in Chapter 1 have properties that provide constructive proof of certain structure theorems. Chapter 3 is about bijections; many interesting and subtle bijections are given to prove non-obvious equicardinality theorems. Chapter 4 concerns the involution principle, a recently formalized approach for proving identities for signed sums (e.g., famous determinant formulas) and other identities that don't at first appear to involve signed sums. After a very useful and current bibliography, the book ends with a lengthy appendix giving actual Pascal programs for the main algorithms of the text.

This book is not easy reading. The writing is spare, including the proofs. Fair chunks of the material are probably graduate material. I include here the material on extremal graph theory in Chapter 2 and the material on Young tableaux in Chapters 3 and 4. Perhaps the fact that the reader is encouraged to get hands-on experience through writing and running computer programs makes the constructs and proofs easier to grasp, but I would have to teach the book to believe it.

This book has several special strengths. It takes the reader right up to some active research areas. The problems are graded (1 = plug-in to 4 = term-project, with suffix C meaning computer oriented). The book contains many elegant proofs, many of which I had not seen before. To my knowledge, this is the first upper-level combinatorics text that strongly exemplifies the algorithmic approach.

For *UMAP Journal* readers it is perhaps important to point out that this book does *not* treat, and does not claim to treat, any applications.

Unfortunately, I don't see that I could use this book for a course. First, although I teach at a very selective institution, I would not use this as the main text except in an honors section. Second, it is intended for a third quarter upper-level combinatorics course (though mostly independent of the second term, graph theory). We have only one upper-level, semester course. True, we also teach "Mathematical Algorithms," but I feel the coverage in this book is too narrow for that; we prefer to head towards complexity theory. However, I intend to put this book on library reserve even for our current algorithms course and perhaps lecture on one or two items from it.

In summary, this book should be in the personal library of everyone who ever teaches combinatorics, but there may be few schools which can use it as an undergraduate text.

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Books on Combinatorics and Graph Theory

EDITOR'S NOTE: As Prof. Maurer points out in his review of the book by Stanton and White, interest in algorithmic mathematics has increased rapidly in the last few years. This has been especially true in combinatorics. It would be difficult to overstate how drastically the role of combinatorial mathematics has increased since 1960. This increase is mostly because of computers; computers are, after all, finite machines. But there has also been renewed interest in non-algorithmic combinatorics. However, if a student is learning graph