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C. D. Cothran

Matthew Joseph Landreman , '03

Michael R. Brown

*Swarthmore College*, [doc@swarthmore.edu](mailto:doc@swarthmore.edu)

W. H. Matthaeus

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## Generalized Ohm's law in a 3-D reconnection experiment

C. D. Cothran, M. Landreman, and M. R. Brown

Physics and Astronomy Department, Swarthmore College, Swarthmore, Pennsylvania, USA

W. H. Matthaeus

Bartol Research Institute, University of Delaware, Newark, Delaware, USA

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[1] We report the measurement of non-ideal terms of the generalized Ohm's law at a reconnection site of a weakly collisional laboratory magnetohydrodynamic plasma. Results show that the Hall term dominates the measured terms; resistive and electron inertia terms are small. We suggest that electron pressure (not measured) supports the observed quasistatic reconnection rate, and that anomalous resistivity, while not ruled out, is not required to account for the results. **Citation:** Cothran, C. D., M. Landreman, M. R. Brown, and W. H. Matthaeus (2005), Generalized Ohm's law in a 3-D reconnection experiment, *Geophys. Res. Lett.*, 32, L03105, doi:10.1029/2004GL021245.

### 1. Introduction

[2] Magnetic reconnection, ubiquitous in space and laboratory plasmas [Parker, 1957; Sonnerup *et al.*, 1981; Priest and Forbes, 2000], is responsible for plasma flows, ion heating, and 3-D topological changes. Classical descriptions of reconnection are in terms of a simple resistive, single fluid magnetohydrodynamics (MHD) model, valid for a weakly collisional plasma only at large scales. At scales comparable to or smaller than the ion inertial length  $c/\omega_{pi}$ , single fluid MHD breaks down and a better model is needed, such as an ion-electron two-fluid model [Biskamp *et al.*, 1995]. A key quantity is the electric field near an X-type neutral line (in 2-D) or a magnetic separator (in 3-D). In low collisionality plasmas the structure of Ohm's law as modified by kinetic effects is of special importance in understanding reconnection. Here we provide experimental results of central relevance to this issue.

[3] A generalized Ohm's law can be written:

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e + \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t} \quad (1)$$

The  $\eta \mathbf{J}$  term may be due to classical collisional resistivity or "turbulent resistivity" due to fluctuations. The Hall term,  $(1/ne)(\mathbf{J} \times \mathbf{B})$ , associated with differential flow of ions and electrons, becomes appreciable at the ion inertial scale  $\rho_{ii} = c/\omega_{pi}$ . The electron pressure tensor term is formally of the order of  $\beta_e \rho_{ii}$  (where  $\beta_e$  is the ratio of electron pressure to magnetic pressure). The final term in equation (1), the electron inertia term is appreciable at the electron inertial scale  $c/\omega_{pe}$ . For "ideal" MHD,  $\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$ , and the magnetic flux is "frozen-in" the bulk plasma moving at the center of mass velocity  $\mathbf{u}$ .

[4] Throughout most of the plasma, we expect substantial inductive electric fields driven by large scale plasma motions. Near reconnection zones, the inductive electric field should give way to non-ideal effects. For weakly collisional plasma such as in the Swarthmore Spheromak Experiment (SSX), the resistive electric field is not large enough to mask other kinetic nonideal terms. The Hall term becomes important within  $c/\omega_{pi}$  of a reconnection site, but in the collisionless limit, only electron pressure, electron inertia, and possibly a term involving turbulent resistivity are available for dissipation of magnetic flux at the neutral line. An understanding of the various electric field contributions near the reconnection zone is potentially of great use in identifying the presence of reconnection and its relation to surrounding dynamical processes.

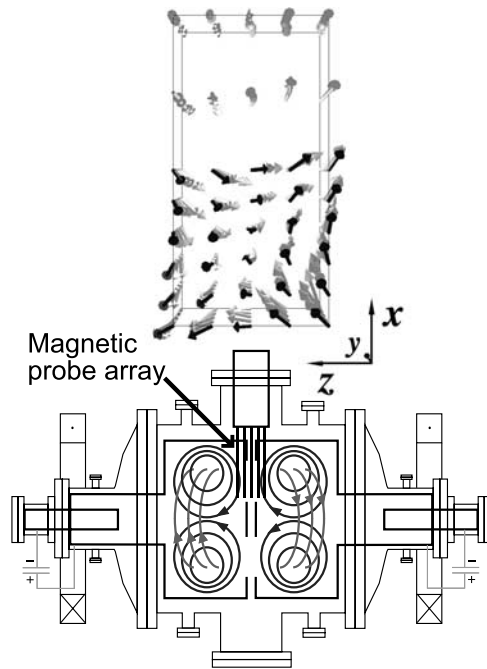
[5] Previously, in a reconnection experiment relevant to electron MHD (unmagnetized ions), Stenzel and Gekelman [Stenzel *et al.*, 1982] measured terms in Ohm's law at the  $c/\omega_{pe}$  scale. There are indications in simulations and spacecraft observations that the Hall effect term in Ohm's law becomes important at the  $c/\omega_{pi}$  scale. Recent magnetospheric data from the POLAR, WIND, Cluster spacecraft are consistent with generalized Ohm's law effects [Øieroset *et al.*, 2001; Mozer *et al.*, 2002; Scudder *et al.*, 2002; Runov *et al.*, 2003]. The quadrupole Hall signature is seen in 2.5D reconnection simulations, and is attributed to the decoupling of electron and ion motion at the  $c/\omega_{pi}$  scale [Shay *et al.*, 1998, 1999; Pritchett, 2001]. Simulations have also shown that electron pressure effects can trigger fast reconnection [Ma and Bhattacharjee, 1996; Kuznetsova *et al.*, 2001].

[6] In this Letter, we present direct laboratory measurements of non-ideal terms that contribute to the generalized Ohm's law in the weakly collisional SSX plasma. The focus is on contributions to Ohm's law that can be evaluated in the reconnection zone using 3-D vector magnetic field probe data.

### 2. Experiment

[7] The Swarthmore Spheromak Experiment (SSX) [Brown, 1999] is designed to study reconnection and self-organization due to the controlled, reproducible interaction of two spheromaks. SSX has recently measured the 3-D magnetic structure of the reconnection region [Cothran *et al.*, 2003] and observed a reconnection-associated energetic particle population [Brown *et al.*, 2002a].

[8] Two coaxial magnetized plasma guns at each end of SSX (Figure 1) produce spheromak plasmas within separate cylindrical shell copper flux conservers (perfectly conducting on the timescale of the experiment). Large sectors are



**Figure 1.** A sketch of the SSX device and a sample of vector magnetic probe data (see text) at  $t = 64 \mu\text{s}$  in an RL merging experiment. See color version of this figure in the HTML.

cut out of the adjacent walls at the midplane to allow the two spheromaks to interact locally, without compromising gross stability (e.g., against tilting). These sectors define the region where reconnection occurs. SSX is unique in that reconnection is remote from the plasma sources and is not directly driven.

[9] SSX produces MHD plasmas. Typical parameters are  $|\mathbf{B}| \approx 500\text{--}1000 \text{ G}$ ,  $T_e \approx T_i \approx 10\text{--}30 \text{ eV}$ , and  $n_e \approx 0.5\text{--}1.0 \times 10^{14}/\text{cm}^3$ , as measured with magnetic probes, ion energy analyzers ( $T_i$ ), a triple Langmuir probe ( $T_e, n_e$ ), and a quadrature laser interferometer (line averaged  $n_e$ ). The Lundquist number is  $S \approx 1000 \gg 1$ . Kinetic length scales  $c/\omega_{pi} \approx 2 \text{ cm}$  and ion gyroradius  $\rho_i \approx 1 \text{ cm}$  are much smaller than the macroscopic scale  $L = 12.5 \text{ cm}$  (the spheromak minor radius). The Alfvén crossing time of distance  $L$  is  $\approx 3 \mu\text{s}$ . The ion mean free path is  $\lambda_{ii} \approx 5 \text{ cm}$ , and  $\omega_{ci}\tau_{ii} \approx 6$ . Therefore, these are weakly collisional, MHD spheromaks.

[10] Each gun can produce a spheromak of either sign (L or R for left- or right-handed) of magnetic helicity. In counter helicity (LR or RL) merging, reconnection activity is greatest since the poloidal and toroidal fields of the two spheromaks are both antiparallel in this configuration. Typical counter-helicity evolution is well understood [Kornack *et al.*, 1998; Brown, 1999; Lukin *et al.*, 2001; Brown *et al.*, 2002b; Cothran *et al.*, 2003]. After the spheromaks eject from the guns at  $t \approx 20 \mu\text{s}$ , there is an initial driven phase lasting until  $t \approx 50 \mu\text{s}$ . A longer period of weakly driven or “spontaneous” reconnection occurs between the now largely relaxed, force-free spheromaks, lasting tens of Alfvén crossing times.

[11] 3-D magnetic measurements are made within the reconnection region (see Figure 1) using a  $5 \times 5 \times 8$  array of vector magnetic probes [Landreman *et al.*, 2003], com-

posed of 25 linear probes, each containing a triplet of B-dot (pick-up) coils at 8 locations, for a total of 200 vector  $\mathbf{B}$  measurements. The full probe array is sampled every  $0.8 \mu\text{s}$  during an experiment, thus resolving MHD fluctuations. The  $2 \text{ cm}$  lattice spacing resolves relevant kinetic length scales, in particular  $c/\omega_{pi}$ . The measurement error is  $20 \text{ G}$  in each field component. Figure 1 includes a sample of magnetic data, clearly showing an X-type reconnection configuration.

[12] These 3-D magnetic field data sets permit construction of all but the electron pressure term on the right hand side of equation (1). Interior (boundary) spatial derivatives are computed at second (first) order on the lattice. Second derivatives are calculated to evaluate terms in the curl of equation (1). For the order of magnitude distinctions described in the next section, the (limited) precision and accuracy of this procedure is sufficient.

### 3. Results

[13] The data presented are from both a representative single merging experiment and an average over an ensemble of 36 such experiments that shared identical external preparation. While Ohm’s law applies generally throughout the reconnection region, we select a lattice point at the center of the probe array for examination. This location is known to be less than one ion inertial length (one lattice spacing) distant from the neutral point. The resistive term is calculated using Spitzer resistivity.

[14] Table 1 presents the magnitudes of three terms in Ohm’s law for both the single experiment and the average data set at two times, when the reconnection is driven ( $t = 37 \mu\text{s}$ ) and spontaneous ( $t = 64 \mu\text{s}$ ). The key result is that the Hall term dominates the resistive and electron inertia terms. The ordering is preserved at other locations within the probe array (not shown). This is so despite the fact that the neutral point is nearby, where the Hall term crosses through zero. The resistive term is  $15\text{--}70$  times weaker than Hall, while the electron inertia is more than four orders of magnitude weaker.

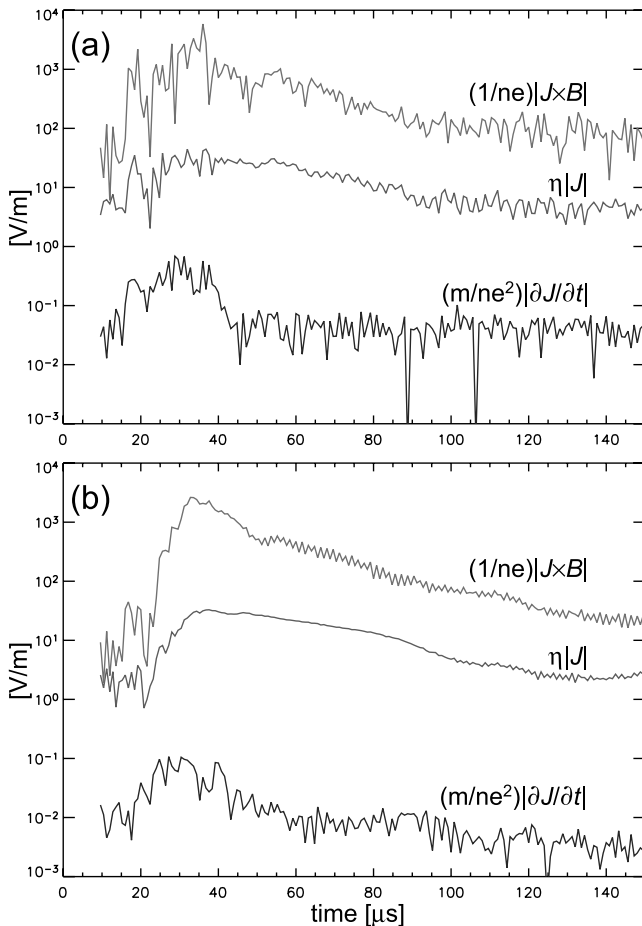
[15] The relative proportion of these terms is maintained throughout a merging event. The time dependence is shown for a single experiment and the averaged data in Figure 2. The flat behavior at late times is a noise floor due to the  $20 \text{ G}$  error and the finite difference derivatives. This is the origin of the error indicated in Table 1.

[16] Note that we do not measure  $n_e$ ,  $T_e$ , or  $T_i$  in the reconnection region so the electron pressure gradient is undetermined. We do not directly measure the total electric field, although the reconnection electric field, deduced by other methods, will be important to the interpretation of

**Table 1.** Magnitude of Terms in Generalized Ohm’s Law From Single Experiment and Average of 36 Experiments<sup>a</sup>

| [V/m]   | One Experiment       |                      | Average Data Set     |                      |      |            |
|---|----------------------|----------------------|----------------------|----------------------|------|------------|
|   | $t = 37 \mu\text{s}$ | $t = 64 \mu\text{s}$ | $t = 37 \mu\text{s}$ | $t = 64 \mu\text{s}$ |      |            |
| $\eta \mathbf{J} $                              | 44                   | 15                   | $\pm 7$              | 33                   | 19   | $\pm 2$    |
| $\frac{1}{ne} \mathbf{J} \times \mathbf{B} $    | 2800                 | 700                  | $\pm 100$            | 2000                 | 330  | $\pm 20$   |
| $\frac{m_e}{ne} \partial\mathbf{J}/\partial t $ | 0.2                  | 0.04                 | $\pm 0.03$           | 0.03                 | 0.01 | $\pm 0.01$ |

<sup>a</sup>Errors reflect only uncertainty arising from field component measurements; no estimate of systematic error due to the accuracy of finite element derivatives is included.



**Figure 2.** Time history of resistive, Hall and electron inertia contributions to Ohm's law: (a) single experiment; (b) average of 36 experiments. See color version of this figure in the HTML.

these results (see below). A related and important measured quantity is  $\partial\mathbf{B}/\partial t = -\nabla \times \mathbf{E}$  which can be used to examine the curl of Ohm's Law. Since  $\nabla \times (\nabla \cdot \mathbf{P}_e)/ne \neq 0$  in general, Faraday's Law becomes:

$$\nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla \times \frac{1}{ne} \nabla \cdot \mathbf{P}_e = \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \eta \mathbf{J} + \nabla \times \frac{1}{ne} (\mathbf{J} \times \mathbf{B}) + \nabla \times \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t} \quad (2)$$

The time dependence of the right hand side terms measured using the averaged data set at the early time ( $t = 37 \mu\text{s}$ ) are displayed in Figure 3. Since the noise floor is higher for these second derivative terms, only the 20 to 40  $\mu\text{s}$  interval is useful. Note that  $\eta$  and  $1/ne$  are assumed to be uniform, so that they commute with  $\nabla \times$ .

[17] Once again, the Hall contribution dominates the terms we can construct. Much smaller are the resistive, the electron inertia, and the  $\partial\mathbf{B}/\partial t$  contributions. The suggestion is that a large inductive term is likely present in order to balance the large Hall contribution to equation (2) and thus satisfy Faraday's law. There also may be a large contribution due to off-diagonal terms in the electron pressure tensor [Scudder *et al.*, 2002]. An equally important conclusion is that the reconnection is evidently occurring quasistatically,

in the sense that the overall magnitude  $|\partial\mathbf{B}/\partial t|$  is much less than some of the individual terms that contribute to it.

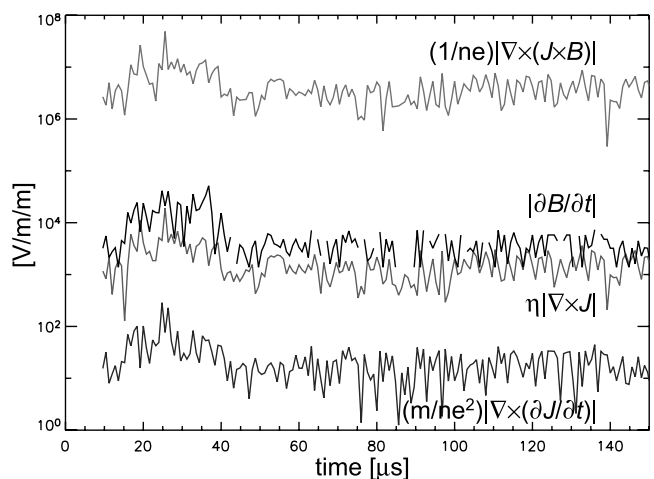
[18] Experimental analysis of both Ohm's Law and Faraday's Law indicate a dominant balance of three terms: Hall effect, induction, and electron pressure contributions. This balance necessarily involves substantial cancellations. Moreover the tradeoff amongst these terms is expected to be spatially independent due to the observed quasi-static conditions. This conclusion is reinforced by similar observations at the other 200 probe locations.

#### 4. Discussion

[19] The rate of reconnection is described quantitatively by the electric field at the separator [Priest and Forbes, 2000]. From a finite set of probes it is improbable that one can determine whether the reconnection is locally of the null point ( $\mathbf{B} \equiv 0$ ) type or the component-type ( $\mathbf{B} \neq 0$ ) at the separator, and it is improbable to expect to decompose the Ohm's Law (as done above) precisely at the null point or separator. However, the value of the reconnection electric field can be determined by other means.

[20] Inspecting equation (1), for neutral point reconnection, the inductive and Hall electric fields vanish at the separator, and only the resistive, electron pressure and electron inertia terms may contribute. For component reconnection, it is the parallel electric field at the separator that is important. Dotting equation (1) with  $\hat{\mathbf{B}}$ , the parallel Ohm's law is  $E_{\parallel} = \eta \mathbf{J}_{\parallel} - \frac{1}{ne} (\nabla \cdot \mathbf{P}_e)_{\parallel} + \frac{m_e}{ne^2} \frac{\partial \mathbf{J}_{\parallel}}{\partial t}$ . Again, the only possible contributors are resistive, electron pressure and electron inertia terms. For the experiments reported in this paper, electron inertia has been measured to be small, and we focus on the other two contributions (we leave open the possibility that there is an unresolved very thin  $c/\omega_{pe}$  layer in which this ordering is incorrect).

[21] Cothran *et al.* [2003] provide an estimate for a lower bound on the reconnection rate based upon observed macroscopic topology change of flux tubes from 32 to 64  $\mu\text{s}$ . They inferred an inflow velocity of a few times  $10^5 \text{ cm/s}$ , equivalent to a reconnection electric field of approximately



**Figure 3.** Time history of the  $\partial\mathbf{B}/\partial t$ , resistive, Hall, and electron inertia contributions to the curl of Ohm's law for a single representative experiment. See color version of this figure in the HTML.

100 V/m. An independent method [Brown *et al.*, 2002a] provides an upper bound. Assuming direct acceleration, analysis of the energy distributions of ions leaving the reconnection region parallel to the neutral line yields a reconnection electric field of about 1000 V/m (this likely holds during the earliest driven phase). Adopting an intermediate value, we ask whether the measured Ohmic electric field is of sufficient strength to account for this reconnection rate, and if not, which terms in Ohm's law might support this electric field.

[22] Based upon either neutral point or component reconnection scenarios, the conclusions are the same, and can be inferred from Figure 2. The electron inertia term is negligible, while the collisional Ohmic term can support at most about 40 V/m near the neutral point (or line). The Hall term is of the order of 1000 V/m for most of the period from 30 to 70  $\mu$ s, but cannot contribute at the neutral point, or along a separator. The remaining possibility is that the reconnection electric field at the neutral point is carried by either turbulent resistivity, or by a large electron pressure contribution. Recall now that we have indications that near the SSX reconnection sites there is both quasi-static conditions and near pressure balance. Also, electron and proton temperatures are similar to one another. On this basis, we deduce that  $\nabla \cdot \mathbf{P}_e \approx \mathbf{J} \times \mathbf{B}$  in this region. It therefore seems most likely to us that the electric field associated with the electron pressure term carries the reconnection electric field at the neutral point. This inference is subject to the caveat that we can extract no information experimentally concerning the tensor structure of  $\mathbf{P}_e$ .

## 5. Conclusion

[23] We have examined electric field contributions to the generalized Ohm's law, by direct experimental evaluation of the associated quantities in the SSX experiment. Throughout the present analysis we have ignored possible contributions to  $\eta$  due to turbulence and fluctuations, which might elevate the resistive contribution to the electric field [Ji *et al.*, 1998]. While we found, where measured, that the electron inertial electric field is small, we cannot rule out that it is much greater in a thin unresolved layer. Higher resolution measurements in time and space could in principle lead to some revision based upon these effects.

[24] With this in mind, we find that the Hall effect, the largest of the measured terms, must be in balance with the inductive and electron pressure terms. The resistive contribution is much smaller, and the smallest observed electric field is that associated with electron inertia. We find that the observed time derivative of the magnetic field is much smaller than the individual terms that contribute to it, so reconnection occurs quasi-statically. Using previous estimates of the reconnection rate we infer that the electron pressure contribution to the electric field at the separator must be comparable to the Hall effect electric field nearby. This is also consistent with quasi-static pressure balance considerations. Substantial cancellation is required to achieve the observed quasi-static condition.

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M. R. Brown, C. D. Cothran, and M. Landreman, Physics and Astronomy Department, Swarthmore College, 500 College Avenue, Swarthmore, PA 19081, USA. (ccothra1@swarthmore.edu)

W. H. Matthaeus, Bartol Research Institute, University of Delaware, Newark, DE 19716-4793, USA.