

Swarthmore College

Works

Engineering Faculty Works

Engineering

4-1-2013

Computer Guided Instruction For Creation Of Root Locus Plots

Erik Allen Cheever , '82

Swarthmore College, erik_cheever@swarthmore.edu

A. Moser

Follow this and additional works at: <https://works.swarthmore.edu/fac-engineering>



Part of the [Engineering Commons](#)

[Let us know how access to these works benefits you](#)

Recommended Citation

Erik Allen Cheever , '82 and A. Moser. (2013). "Computer Guided Instruction For Creation Of Root Locus Plots". *Computers In Education Journal*. Volume 23, Issue 2.

<https://works.swarthmore.edu/fac-engineering/84>

This work is brought to you for free by Swarthmore College Libraries' Works. It has been accepted for inclusion in Engineering Faculty Works by an authorized administrator of Works. For more information, please contact myworks@swarthmore.edu.

COMPUTER GUIDED INSTRUCTION FOR CREATION OF ROOT LOCUS PLOTS

Erik Cheever, Allan Moser
Engineering Department
Swarthmore College

Abstract

The creation of root locus plots is widely taught as part of introductory Control Theory courses, but it is difficult for students to learn. Typically, students are taught a set of rules to apply to a transfer function so they can sketch the resulting root locus diagram. This paper describes a MATLAB® program, RLocusGui, which helps students learn the techniques and sketch these plots. The program takes a transfer function and applies the same rules used when making a root locus sketch by hand. While doing so, it not only explains how the rule is applied to the transfer function at hand, but also graphically depicts the result. If desired, the program creates a web page that describes, with text and graphics, how each of the rules is used, in turn, to generate the root locus plot.

Introduction

Root locus plots [1] are one of the fundamental tools used to analyze and design Control Systems. Introductory Control Theory texts [2, 3] typically have at least one chapter devoted to their creation and use. Many computer programs exist (e.g., MATLAB) that will accurately create root locus diagrams given a system transfer function. While these diagrams are accurate, they don't provide any information about how their shape is related to the transfer function. However, students must fully understand this relationship so they can determine how to use the diagram to design a controller that achieves satisfactory performance. For this to be possible, students must know how the transfer function affects the shape of the root locus. These effects are taught through a set of rules used to sketch, by hand, the root locus diagram for a given transfer function. Drawing these diagrams entails a

process that is often difficult to learn, as well as to teach.

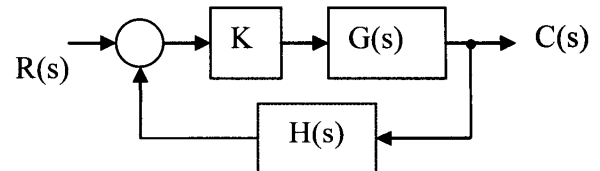


Figure 1: A generic closed loop system.

The process of sketching root locus plots centers around a set of rules, and starts with the loop gain. A generic single-loop transfer function is shown in Figure 1. The loop gain transfer function, $L(s)=G(s)H(s)$, is defined as the ratio of two polynomials in s ; $L(s)=N(s)/D(s)$ where $N(s)$ is the numerator polynomial and $D(s)$ is the denominator polynomial. There is a single gain, K , that is adjusted. A subset of these rules reads:

1. The locus starts at the poles of $L(s)$ ($K=0$) and ends at its zeros (as $K\rightarrow\infty$), including any zeros as $|s|\rightarrow\infty$.
2. The locus is symmetric about the real axis.
3. The number of branches of the locus equals the order of $D(s)$.
4. The locus exists on the real axis to the left of an odd number of poles and zeros of the transfer function.

This set is not exhaustive nor will the rules be derived here; they can be found in introductory Control Theory texts [2, 3].

To the student, difficulty arises because many of the rules seem arcane and perhaps arbitrary. Part of this difficulty results from the relative paucity of examples. Often the student has access only to a few examples from the text, and perhaps one or two more from class notes. Compounding this last difficulty is the fact that many students are "visual" learners according to Felder's index of learning styles [4] who would

benefit from more drawings. The ability to generate additional sketches, along with a description of how the rules are applied, can reinforce learned concepts, and allow the student to investigate a broad variety of “*What if ...?*” scenarios. A root locus expert system has been developed [5], but it requires specialized software, and chooses from among a set of transfer functions (limited to third order). As noted previously, MATLAB makes a very accurate plot of the root locus diagram, but does this using brute-force numerical solutions. It does nothing to describe how the pertinent rules could be applied to make a sketch by hand. This paper describes software that overcomes these limitations and makes students active participants in the process of learning, rather than simply observers. Kolb’s theory of experiential learning posits that experience, which can be provided by more examples, and reflection are part of student learning [6].

To the teacher, the difficulty in teaching root locus diagrams is twofold. First, there is the conceptual problem of the derivation of the rules used to sketch the root locus, and a description of just what the root locus diagrams represent. This paper does not address this issue. The second problem, however, is more mechanical and thus amenable to solution by computer; it is difficult to sketch root locus diagrams by hand. Drawing any graph by hand, whether it be on a chalkboard, whiteboard or on an overhead transparency, is, by its very nature, inaccurate. Paper handouts with examples worked out can be useful. They are not, however, generally useful for answering a student’s “*What if ...?*” questions, due to their static nature.

This paper describes a software tool designed to meliorate difficulties of students and of teachers. The tool is a GUI (Graphical User Interface) written in MATLAB that takes as input a loop gain transfer function. The program then determines how each root locus rule is applied, and gives a thorough description of how the particular rule is applied to the given transfer function. Where appropriate, the

program also visually depicts the rule as applied to the given transfer function. If desired, a web page can be created that describes, in turn, the application of each rule. The software can be downloaded from the MathWorks web site [7].

The Software

A description of the operation of the software follows, and then it is illustrated with an example. Due to the length of the code, over one thousand lines, no detailed explanation of the software is given, but the source code is available for inspection.

The software applies the following 8 rules to sketch the root locus.

1. The root locus is symmetric about the real axis.
2. The number of branches of the locus equals the order of $D(s)$.
3. The locus starts at the poles of $L(s)$ ($K=0$) and ends at its zeros (as $K \rightarrow \infty$), including any zeros as $|s| \rightarrow \infty$.
4. The locus exists on the real axis to the left of an odd number of poles and zeros of the transfer function.
5. If $L(s)$ has zeros as $|s| \rightarrow \infty$, there are asymptotes located at the angles $\theta_a = r180^\circ/q$, where q equals the number of these zeros, and $r = \pm 1, \pm 3 \dots$ Furthermore, these asymptotes intersect the real axis at $\sigma = (\sum(\text{poles}) - \sum(\text{finite zeros}))/q$.
6. The break-away (or break-in) points along the real axis, if there are any, occur where $N(s)D'(s) - N'(s)D(s) = 0$.
7. The angle of departure when $K=0$ from a complex pole, p_i , of $L(s)$ is given by

$$\theta_{\text{depart}, p_i} = 180^\circ + \sum_j \theta_{z_j} - \sum_{j \neq i} \theta_{p_j}$$

where θ_{z_j} is the angle between the zero of $L(s)$, z_j , and the pole, p_i , and θ_{p_j} is the angle between the pole of $L(s)$, p_j , and p_i .

8. The angle of arrival as $K \rightarrow \infty$ at a complex zero, z_i , of $L(s)$ is given by

$$\theta_{\text{arrive}, z_i} = 180^\circ - \sum_{j \neq i} \theta_{z_j} + \sum_j \theta_{p_j}$$

where θ_{zj} is the angle between the zero of $L(s)$, z_j , and the zero, z_i , and θ_{pj} is the angle between the pole of $L(s)$, p_j , and z_i .

In addition, the software also

9. Determines and displays pertinent information about $L(s)$ (i.e., location and number of poles and zeros...)
10. Finds where the locus crosses the imaginary axis, if anywhere.
11. Demonstrates how to find pole locations on the axis given a value of the gain, K .
12. Demonstrates how to find the value of K , given the location of a closed loop pole.

At this point it is worth reiterating that if any of these rules are unclear to the reader, an introductory Control Theory text should be consulted.

An Example to Demonstration

The software is invoked via the MATLAB command *RLocusGui(Sys)* where *Sys* is a transfer function object in the MATLAB environment. The program first determines all necessary information about the loop gain transfer function, $L(s)=G(s)H(s)$, necessary for making a sketch. This includes the number and location of poles and zeros of $L(s)$, and which poles and zeros are complex. The most relevant information is then displayed, along with the completed root locus. This is best explained with an example.

Consider the transfer function

$$L(s) = \frac{s+2}{s^3+5s^2+4s}$$

The software is invoked as follows:

```
>> Sys=tf([1 2],[1 5 4 0]);
>> RLocusGui(Sys)
```

The GUI initializes and then displays a window as shown in Figure 2 (next page). The diagram has been annotated to show its

constituent parts. Starting in the upper right, they are:

- A. The transfer function, $L(s)=G(s)H(s)$.
- B. The completed root locus. Note that there is an optional check box to display a grid (constant ζ and ω_n).
- C. Radio buttons to allow for selection of rule to be displayed.
- D. A description of the rule (selected by radio button in box "C") as applied to the specified transfer function.
- E. A button that will open a web-page that gives a detailed description of the chosen rule, both its derivation and its application. Internet access is required.
- F. A button that creates and opens a web-page that describes, in text and graphically, each rule.
- G. A button that goes to a web-page that describes the root locus technique in detail. Internet access is required.
- H. A button that closes the GUI.

Several of these items are not obvious, and are described in more detail below.

Initially the box "D" shows some generic information about the transfer function that will be used to generate the root locus plot: the location of poles and finite zeros, the number of zeros as $|s| \rightarrow \infty$, the numerator and denominator polynomial, and the characteristic equation.

To select a different rule the user simply selects a different radio button. For example, to explore the behavior of the locus as $K \rightarrow \infty$, the sixth button is selected. Doing so changes the window so that it appears as in Figure 3. Much remains the same, but there are two significant differences. First, the text in the box "D" has changed to describe how the rule is applied to the specified transfer function. Second, a new set of axes has appeared that graphically demonstrates the rule. On a computer screen the diagram is in color, so the highlighting is more obvious.

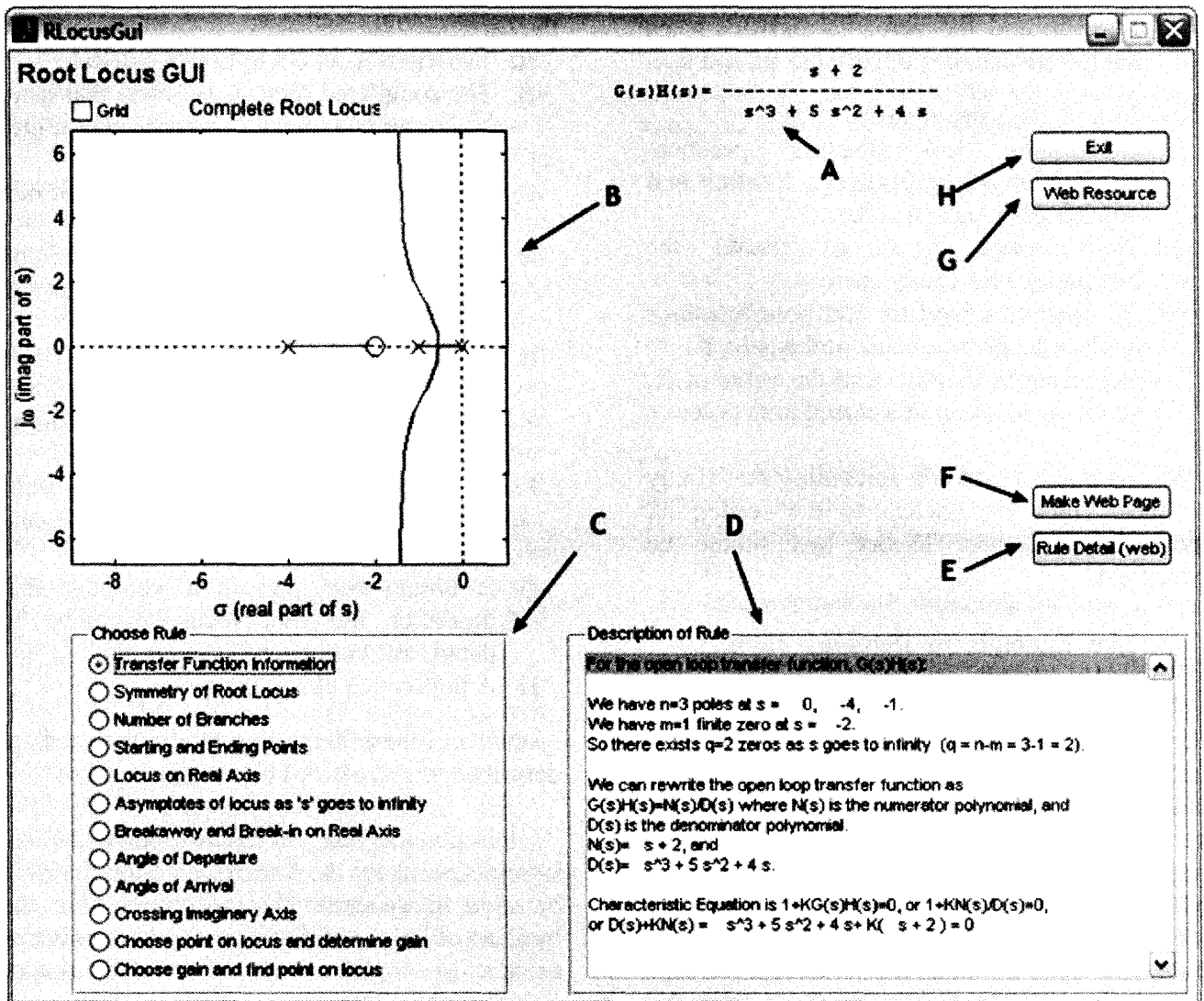


Figure 2: Initial window shown by *RLocusGui* software. See text for descriptions of annotations.

Each rule with a radio button may be similarly chosen. The text in box “D” changes accordingly, and if appropriate the second set of axes appears and demonstrates the specified rule. The second (symmetry) through eighth (angle of arrival) rule perform as expected based on descriptions in standard Control Theory texts, and will not be elaborated upon. The last three buttons behave somewhat differently, and are described in the following three paragraphs.

Crossing Imaginary Axis: Standard texts teach that the location at which the locus crosses the

imaginary axis is determined by using the Routh criterion to find the value of K for which a system becomes marginally stable, and then uses that value to find the frequency of oscillation. The Routh criterion is hard to implement computationally, so the *RLocusGui* program simply searches the loci for locations where it goes from the left-half-plane to the right-half (or vice-versa), and then interpolates to find the value of the gain at that point, as well as the location where the imaginary axis is crossed.

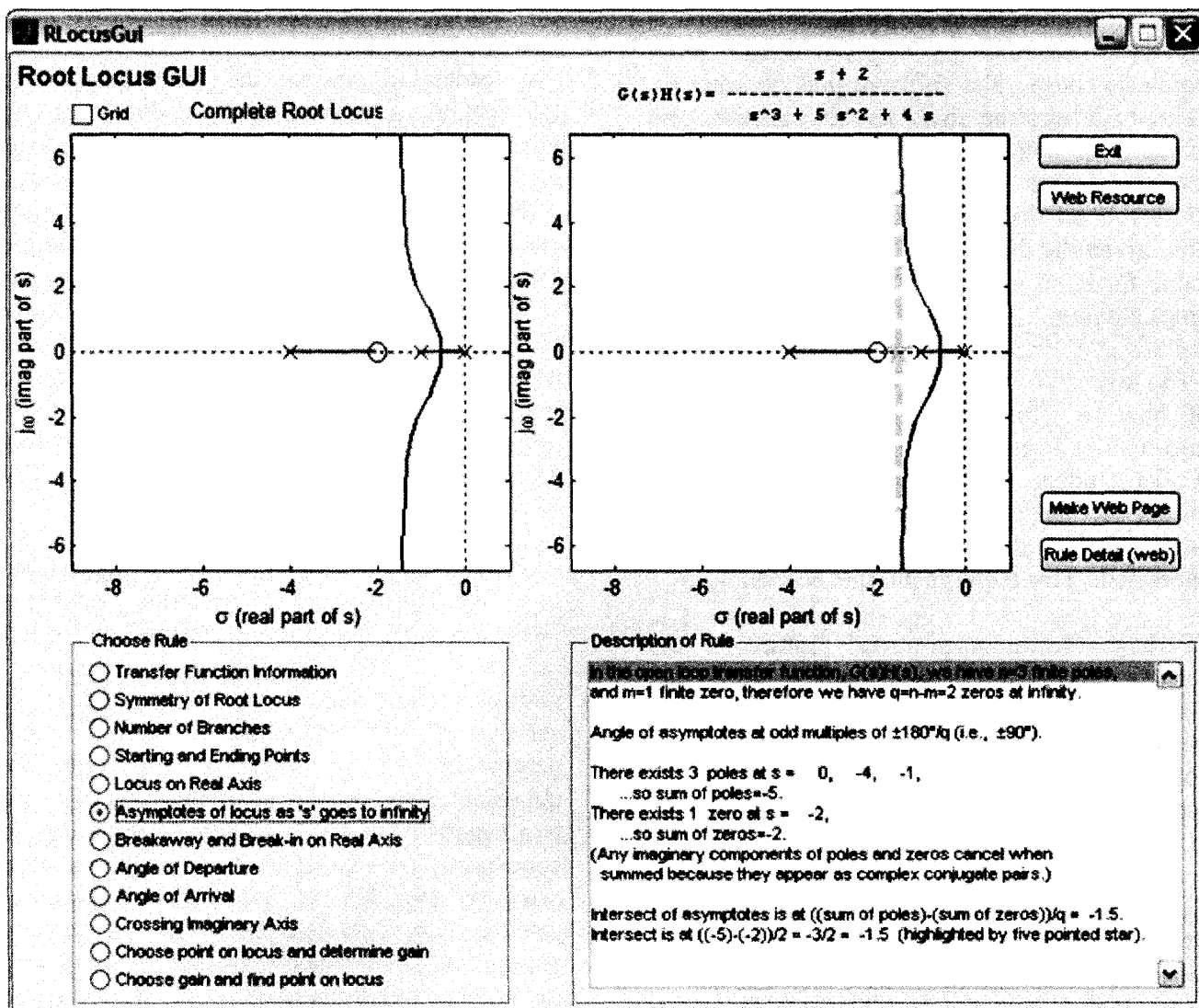


Figure 3: This window shows the same system as Figure 2, but demonstrates system behavior as $|s| \rightarrow \infty$. Note: a different radio button is chosen in box "C"; the text in box "D" is changed; another set of axes has appeared to illustrate the rule. Some information is more clearly shown when displayed in color.

Choose point on locus and determine gain: This is similar to the MATLAB function "rlocfind." When this radio button is chosen, the software tries to find a spot on the locus, s_1 , where $\zeta \approx 1/\sqrt{2}$ (this is more-or-less arbitrary, but was chosen because controllers are often designed near these points). The resulting value of K is then determined from the relationship $K = -D(s_1)/N(s_1)$, with the process described in box "D." To make things a bit more realistic, the program actually chooses a spot near the locus, but not precisely on it (as would be done if working by hand and examining a root locus

sketch). The resulting value of K is complex with a small imaginary part that can be ignored. At this point, the user can choose to specify another location on the graph, and the program calculates the resulting value of gain, K . This is also very similar to the behavior of the "rlocfind" command, except that the *RLocusGui* program does not snap the selected spot to the locus, and if the specified root location is not exactly on the locus, the resulting value of K is complex.

Choose a gain and find point on locus: This last selection uses the fact that the characteristic

equation of the closed loop transfer function is given by $D(s) + KN(s) = 0$. When this radio button is chosen, the software puts an arbitrary value of K into the characteristic equation, and explains (in box “D”) how the resulting closed loop poles are found. These poles are also displayed on the root locus plot. The user is then given the opportunity to enter his/her own value for K , at which time the new closed loop poles are found.

The text in box “D” is a very brief description of how to apply the chosen rule, with no information at all about the genesis of the rule. If the student desires such an explanation, hitting the “Rule Detail (web)” button opens a browser that describes how the rule was generated. This requires internet access.

Perhaps the most powerful feature of the software allows the user to create a web page, complete with text and graphics, which explains how a hand sketch would be made. The page applies each rule in turn and sends any text that would be generated in box “D” to the web page. In addition, any graphs that appear in the second set of axes are also sent to the web page. After all of the rules have been applied, and the entire web page completed, a browser opens to display the page. A sample of part of one of the web pages that shows the results from the radio buttons up to the seventh one, break-away and break-in points, is given in Appendix 1. The text and graphics from the other rules were deleted from the appendix to save space, and the page was reformatted slightly for the same reason. The full web address is given in the figure caption. The web page is similar in content to examples of the root locus that appear in textbooks, but it is tailored to the transfer function specified by the user. In this way a student may pick any transfer function and quickly determine not only what the sketch looks like (MATLAB’s “rlocus” command does that extremely well) but, more importantly, *why* it looks as it does.

Assessments

In order to assess the effectiveness of RLocusGui, a survey was conducted in an introductory control theory class with ten students. The survey was administered using a web-based tool, SurveyMonkey [8], which provided a convenient platform for presenting the questions and collecting responses, as well as maintaining anonymity. The students taking the survey previously had a lecture and homework assignment on root locus design. Subsequently, they were given an entire class period (75 minutes) to freely explore RLocusGui and complete the survey. The survey is shown in Appendix 2.

The survey was designed using suggestions from the SurveyMonkey web-site's reference "Best Practices for Survey Design" and other references [9, 10]. Questions were constructed with a monadic scale which is appropriate for surveys designed to measure attitudes and perceptions [9]. In order to avoid bias, the questions were bipolar with two positive and two negative categories. There is some debate concerning the addition of a middle neutral category [9, 10]. Eliminating the neutral category forces an opinion which may not be appropriate if responders are not familiar with the content being assessed. However, for this survey, students had ample prior knowledge of the material and so were assumed to have an opinion. Thus, we believe the responses would be more informative if the neutral category was eliminated. Additionally, to avoid "bland" responses, the anchor points were more strongly worded (e.g. "very useful" ... "difficult"). Additionally, we tried to phrase questions in an uncritical way to avoid positive bias. For example, opposites of "very useful" and "not useful at all" were used as opposed to "very useful" and "useless."

The survey consists of an introduction and directions for the survey followed by nine questions. The first two questions measure the student's opinion as to the ease of use of RLocusGui and their motivation to use it. These questions are useful in the interpretation of

answers to subsequent questions, since if the student found it difficult to use the tool or was not motivated, one might expect more negative responses regarding specific applications. The third question is a follow-up to the second question; assuming that they believe such a tool might be useful, how useful did they actually find it for exploring examples. The fourth and fifth questions ask about specific features of the tool; namely, how useful did they find the options to examine the root locus rules and see explanations for each rule. The sixth question compares the utility of the computer-based tool to standard class room learning methods. Question seven, the last question with categorical answers, provides a summary of the student's overall feelings about the learning effectiveness of RLocusGui. Question eight provides an opportunity for an essay response concerning the effectiveness of RLocusGui. The last question provides an open-ended forum for any other comments or suggestions.

Table 1 summarizes the results of the survey. For this table, the categorical responses (e.g., "Very useful," "Somewhat useful," "Not very useful," and "Not useful at all") were converted to numerical values with 1 being the most positive response, and 4 the least positive response. The average values for questions one through five ranged from 1.10 to 1.50, with low variance, meaning all students viewed these aspects of RLocusGui in a very positive sense. Question six, comparing the effectiveness of RLocusGui to standard classroom teaching methods, had an average value of 2.30 with a larger variance. While this is a positive response, there was less consensus. From the complete survey results, shown in Appendix 3, six students felt that RLocusGui was much more effective or somewhat more effective than standard teaching methods, three students felt that it was comparable, and one student felt that it was less effective. Since a lecture and homework assignment had already been devoted to root locus before survey, this result may reflect the degree to which students felt they already understood the subject. The responses to question seven, asking if they felt that

RLocusGui could enhance their understanding, were again all positive with two students responding "very much" and eight somewhat. Complete survey results are in Appendix 3.

Table 1: Averages and variances for the ten students' responses to the assessment survey. The number 1 is the most positive response, and 4 the least positive response.

	Ave	σ
1. How easy was it to use RLocusGui?	1.10	0.32
2. In your opinion, how much do you believe that being able to explore a variety of examples will improve your understanding of the rules for drawing and interpreting root locus diagrams?	1.50	0.53
3. How useful do you find RLocusGui in enabling you to explore a variety of examples?	1.50	0.53
4. How helpful is the option to choose and examine root locus rules individually ("Choose Rule" buttons) and see the explanation of that rule in the "Description of Rule" panel?	1.50	0.53
5. How useful do you find the animated or interactive plots activated with the selection of some rules, or choice of a locus point or gain?	1.40	0.52
6. How effective do you believe RLocusGui is for facilitating learning compared to the lectures and text?	2.30	0.95
7. Indicate how much you feel that using this tool can enhance your understanding of the root locus procedure.	1.80	0.42

Limitations of Software

The software does have some known limitations. It cannot display the complementary root locus (though it can be displayed by inverting the sign of the transfer function: $R_{LocusGui}(-Sys)$). All axes are scaled on the assumption that magnitude of pole and zero locations are on the order of one to ten; it has not been tested with much smaller, or larger, poles and zeros. This is not a serious limitation as most examples in Control Theory texts fall into this category, and those that do not can be scaled so that they do. Please contact the authors if you find any bugs or have suggestions for improvement.

Conclusions

The RLocusGui program is a versatile program for teaching and learning to sketch root locus diagrams by hand.

For teachers, it offers the ability to generate multiple examples in class, quickly display how the individual rules are used, and how they are applied to the resultant plots.

For students, this software gives them the ability to generate multiple examples to help in the learning process. He or she can take examples from class or from a textbook and reproduce them. The students can also break down any example they find, or create themselves, so each rule can be explained in turn. Moreover, they may do so at their own pace.

References

1. W.R. Evans, "Graphical Analysis of Control Systems," in *Trans AIEE*, vol. 67, no. 1, 1948.
2. C.L. Phillips and J.M. Parr, *Feedback Control Systems*, 5th Edition, Prentice Hall, Upper Saddle River, NJ, 2011.
3. N.S. Nise, *Control Systems Engineering*, 4th Edition, John Wiley & Sons, Hoboken, NJ, 2004.
4. R. M. Felder and L. K. Silverman "Learning Styles and Teaching Styles in Engineering Education," *Engineering Education*, 78:7, 1988, 674-681.
5. J. J. Blakely, "An Expert System Root Locus Plotter," in *Int. J. Elect. Eng. Educ.*, vol. 36, 1999
6. D. A. Kolb, *Experiential Learning: Experience as the Source of Learning and*

Development, Prentice Hall, Upper Saddle River, NJ, 1984.

7. Anon, MATLAB Central File Exchange, <http://www.mathworks.com/matlabcentral/fileexchange/14117-gui-for-root-locus>, accessed June 2011.
8. Anon, <http://www.surveymonkey.com/>, SurveyMonkey.com, LLC, Palo Alto, California, USA, accessed August 2011.
9. R. A. Peterson, *Constructing Effective Questionnaires*, 1st edition, Sage Publications, Inc, 1999.
10. G. Iarossi. *The Power of Survey Design*, World Bank Publications, 1 edition, 2006.

Biographical Sketches

Erik Cheever is a professor of engineering at Swarthmore College. He received his BS in engineering from Swarthmore College and his MS and PhD in Bioengineering from the University of Pennsylvania. Professor Cheever's teaching interests include circuits, analog electronics, linear systems and signal processing. His research interests include using embedded microcontrollers for smart appliances and genetic algorithms for analog circuit design. His email is erik_cheever@swarthmore.edu.

Allan Moser is a visiting associate professor at Swarthmore College and also works as a consultant for computational biology and high performance computing. He received his BS in Physics and BA in Mathematics from the University of Texas at Austin, and his MS and PhD in Physics from Purdue University. Dr. Moser's teaching interests include engineering mathematics, modeling of dynamical systems, signal and image processing, and control theory. His research interests include machine learning, pattern discovery, image and signal processing, computational biology, and high performance computing.

Appendix 1: Example Web Page

A portion of the web page generated by the program for the specified transfer function is included below. It was slightly reformatted, and truncated, to save space for publication. The full, unaltered (and color) web page is available at http://lpsa.swarthmore.edu/Root_Locus/RLocusPaper/RLocus.html.

Transfer function

$$G(s)H(s) = \frac{s + 2}{s^3 + 5s^2 + 4s}$$

Xfer Function Info

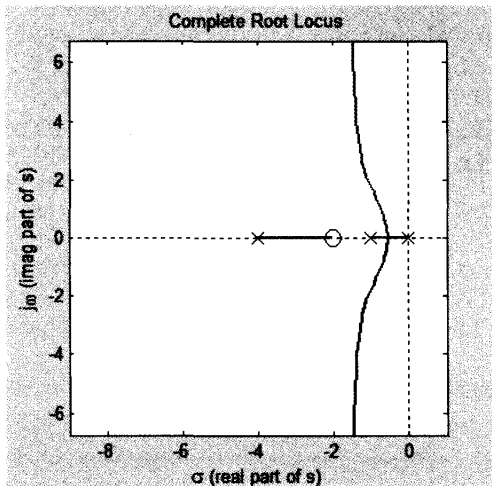
For the open loop transfer function, $G(s)H(s)$: We have $n=3$ poles at $s = 0, -4, -1$. We have $m=1$ finite zero at $s = -2$. So there exists $q=2$ zeros as s goes to infinity ($q = n-m = 3-1 = 2$).

We can rewrite the open loop transfer function as $G(s)H(s)=N(s)/D(s)$ where $N(s)$ is the numerator polynomial, and $D(s)$ is the denominator polynomial.

$N(s) = s + 2$, and $D(s) = s^3 + 5s^2 + 4s$.

Characteristic Equation is $1+KG(s)H(s)=0$, or $1+KN(s)/D(s)=0$,
or $D(s)+KN(s) = s^3 + 5s^2 + 4s + K(s + 2) = 0$

Completed Root Locus



Root Locus Symmetry

As you can see, the locus is symmetric about the real axis

Number of Branches

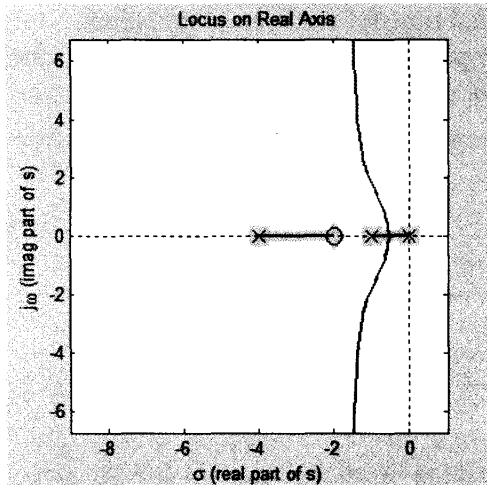
The open loop transfer function, $G(s)H(s)$, has 3 poles, therefore the locus has 3 branches. Each branch is displayed in a different color.

Start/End Points

Root locus starts ($K=0$) at poles of open loop transfer function, $G(s)H(s)$. These are shown by an "x" on the diagram above

As K goes to infinity the location of closed loop poles move to the zeros of the open loop transfer function, $G(s)H(s)$. Finite zeros are shown by a "o" on the diagram above. Don't forget we have we also have $q=n-m=2$ zeros at infinity. (We have $n=3$ finite poles, and $m=1$ finite zero).

Locus on Real Axis



The root locus exists on real axis to left of an odd number of poles and zeros of open loop transfer function, $G(s)H(s)$, that are on the real axis. These real pole and zero locations are highlighted on diagram, along with the portion of the locus that exists on the real axis.

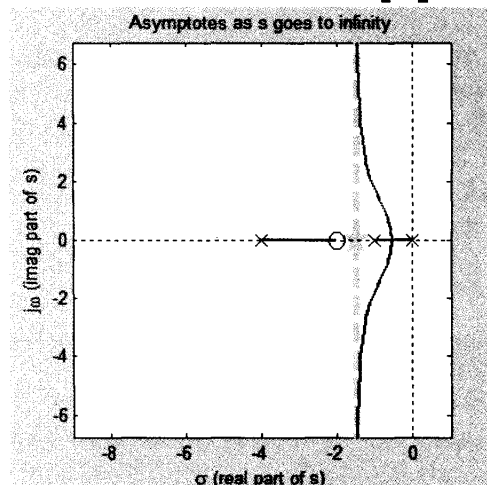
Root locus exists on real axis between:

0 and -1

-2 and -4

... because on the real axis, we have 3 poles at $s = -1, -4, 0$, and we have 1 zero at $s = -2$.

Asymptotes as $|s|$ goes to infinity



In the open loop transfer function, $G(s)H(s)$, we have $n=3$ finite poles, and $m=1$ finite zero, therefore we have $q=n-m=2$ zeros at infinity.

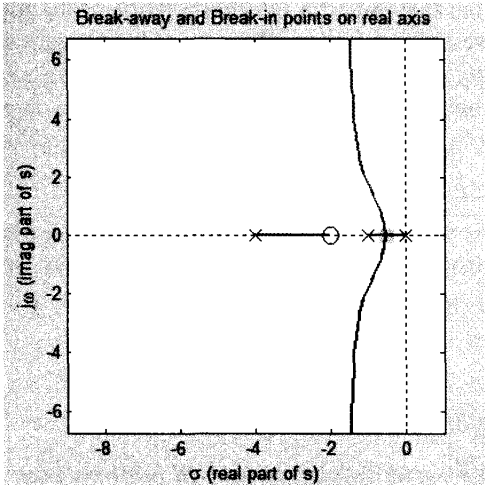
Angle of asymptotes at odd multiples of $\pm 180^\circ/q$ (i.e., $\pm 90^\circ$).

There exists 3 poles at $s = 0, -4, -1$, ...so sum of poles $= -5$. There exists 1 zero at $s = -2$, ...so sum of zeros $= -2$. (Any imaginary components of poles and zeros cancel when summed because they appear as complex conjugate pairs.)

Intersect of asymptotes is at $((\text{sum of poles}) - (\text{sum of zeros}))/q$.

Intersect is at $((-5) - (-2))/2 = -3/2 = -1.5$ (highlighted by five pointed star).

Break-Out and In Points on Real Axis



Break Out (or Break In) points occur where $N(s)D'(s) - N'(s)D(s) = 0$, or $2s^3 + 11s^2 + 20s + 8 = 0$. (details below*)

This polynomial has 3 roots at $s = -2.5 \pm 1.1j$, -0.55 .

From these 3 roots, there exists 1 real root at $s = -0.55$.

These are highlighted on the diagram above (with squares or diamonds.)

These roots are all on the locus (i.e., $K > 0$); and are highlighted with squares.

* $N(s)$ and $D(s)$ are numerator and denominator polynomials of $G(s)H(s)$, and the tick mark, ', denotes differentiation.

$$N(s) = s + 2$$

$$N'(s) = 1$$

$$D(s) = s^3 + 5s^2 + 4s$$

$$D'(s) = 3s^2 + 10s + 4$$

$$N(s)D'(s) = 3s^3 + 16s^2 + 24s + 8$$

$$N'(s)D(s) = s^3 + 5s^2 + 4s$$

$$N(s)D'(s) - N'(s)D(s) = 2s^3 + 11s^2 + 20s + 8$$

Here we used $N(s)D'(s) - N'(s)D(s) = 0$, but we could multiply by -1 and use $N'(s)D(s) - N(s)D'(s) = 0$.

Angle of Departure

No complex poles in loop gain, so no angles of departure.

⋮

and so on

Appendix 2: The Survey Used in this Study

Root locus plots have been covered in class and in the textbook. You have also had a few homework problems asking you to apply the rules for sketching root locus diagrams. However, we believe that in order to gain a deeper understanding of the rules and apply them to create and interpret diagrams, you need to work through many examples and have an opportunity to modify problems to see how they affect results. RLocusGui is a teaching tool that makes it easy to do this. The purpose of this survey is to gauge the effectiveness of RLocusGui in enhancing your understanding of the method.

For this survey, we would like for you to explore this tool using problems you have already solved for homework, examples from the web-page or book, or any other feedback control systems that you are interested in investigating. After you have worked through a few examples, please answer the questions in this survey to help us assess the effectiveness of RLocusGui.

1. How easy was it to use RLocusGui?

- ☐ Very easy ☐ Moderately easy ☐ Somewhat difficult ☐ Difficult

2. In your opinion, how much do you believe that being able to explore a variety of examples will improve your understanding of the rules for drawing and interpreting root locus diagrams?

- ☐ Very much ☐ Somewhat ☐ Not very much ☐ Not at all

3. How useful do you find RLocusGui in enabling you to explore a variety of examples?

- ☐ Very useful ☐ Somewhat useful ☐ Not very useful ☐ Not useful at all

4. How helpful is the option to choose and examine root locus rules individually (the "Choose Rule" buttons) and see the explanation of that rule in the "Description of Rule" panel?

- ☐ Very helpful ☐ Somewhat helpful ☐ Very helpful ☐ Very helpful

5. How useful do you find the animated or interactive plots that are activated with the selection of some rules, or choice of a locus point or gain?

- ☐ Very useful ☐ Somewhat useful ☐ Not very useful ☐ Not useful at all

6. How effective do you believe RLocusGui is for facilitating learning compared to the lectures and text?

- ☐ Much more effective ☐ Somewhat more effective ☐ Comparable ☐ Less effective

7. Indicate how much you feel that using this tool can enhance your understanding of the root locus procedure?

- ☐ Very much ☐ Somewhat ☐ Not very much ☐ Not at all

8. Please add any comments you may have concerning the effectiveness of RLocusGui in contributing to your understanding of the root locus procedure.

9. Do you have any other comments, or can you suggest any improvements to RLocusGui?

Appendix 3: Complete RLocusGui Assessment Results

Response to survey items 1 through 7:	1	2	3	4	5	6	7	8	9	10	Ave	σ
1. How easy was it to use RLocusGui?	1	1	1	1	1	1	1	1	2	1	1.10	0.32
2. In your opinion, how much do you believe that being able to explore a variety of examples will improve your understanding of the rules for drawing and interpreting root locus diagrams?	2	2	1	1	1	1	2	1	2	2	1.50	0.53
3. How useful do you find RLocusGui in enabling you to explore a variety of examples?	1	1	2	2	1	2	1	2	2	1	1.50	0.53
4. How helpful is the option to choose and examine root locus rules individually (the "Choose Rule" buttons) and see the	2	1	1	1	2	1	2	2	2	1	1.50	0.53

explanation of that rule in the "Description of Rule" panel?												
5. How useful do you find the animated or interactive plots that are activated with the selection of some rules, or choice of a locus point or gain?	1	2	2	2	1	1	1	1	1	2	1.40	0.52
6. How effective do you believe RLocusGui is for facilitating learning compared to the lectures and text?	1	2	2	3	3	1	4	3	2	2	2.30	0.95
7. Indicate how much you feel that using this tool can enhance your understanding of the root locus procedure?	2	2	1	2	1	2	2	2	2	2	1.80	0.42

Response to item 8: "Please add any comments you may have concerning the effectiveness of RLocusGui in contributing to your understanding of the root locus procedure."												
1	A brief tutorial would be helpful in order to understand the capabilities of the code.											
2	It is great that it actually explains why the plot looks the way it does. I can use it to check my work and better understand my mistakes.											
4	Seems like an effective teaching tool. It is a nice compliment to the rlocus command because it explains step by step the reasons behind the plot. This could be a very helpful tool for checking homework problems and helping you understand your mistakes.											
5	This did not help with the procedure, in the sense of cranking through making a root locus. But, once I had a sense of the rules, this was very very helpful.											
6	Extremely powerful tool - rules are often quite abstract and hard to connect to actual system behavior. (Disclaimer - I already know root-locus, and thus have some preexisting familiarity with the topic)											
7	Because we have already learned root locus, I wasn't sure how effective the program was at explaining the subject. It was very useful for being able to view different results for varying transfer functions.											
8	RLocusGui will be very helpful as a supplement to learning the rules of root locus construction in a lecture format. The Gui will also be useful when solving particularly difficult problems involving root loci.											
10	The strict derivations on the website are useful, but it would be nice to have a more intuitive explanation of the more complicated rules such as the Angle rules (which are the only ones that were not well covered otherwise). It's a nice tool for demonstrating and exploring what we already sort of learned, and so is an excellent course supplement. It's also nice to be able to easily rigorously graph the root locus plot.											

Response to item 9. "Do you have any other comments, or can you suggest any improvements to RLocusGui?"												
1	I think it would make it easier to see the variety of examples if we were able to change the transfer function on the GUI instead of going back to matlab. Also, 'description of rules' window can walk through how the plot is constructed with steps that are building the animation. For instance, each rule can be depicted on the plot by highlighting.											
5	The "summary of root locus" is silly. Also, when you do $1/s^2$, it messes up on crossing the jw axis. The angle of arrival/departure is complicated and doesnt make much sense. Reloading the animation without clicking twice would be nice											
6	- Allow TF input within GUI - really annoying to have to put in new TF and re-run GUI every time. - Show what whole transfer function looks like as well as $G(s)H(s)$, operating under the assumption that the system is unity-gain feedback and the transfer function is completely modeled as $G(s)H(s)/(1 + G(s)H(s))$ - Error with $(-s - 1)/(s - 1)$ (we think).											
7	The rules were kind of thrown at you all at once. A step by step tutorial on how to find the data to draw the graph might have been more useful than simply giving the equations and the answers right away.											
10	2 Errors Found (These trigger errors in MATLAB) 1) The "Choose Gain and Find Point on Locus" tab triggers an NaN/Inf error when the K slider is brought to the top of the scale. 2) In a transfer function with $m > n$ (Higher order numerator than denominator) and complex zeroes, an error is thrown on selection of the Angle of Arrival tab. Other Comments -It's hard to see the details of complicated graphs. It'd be nice to be able to zoom in/out. -Selecting the Starting/Ending Points tab and then any other tab will overwrite the new tab's plot with the rest of the Start/Endpoint animation											