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Liquid Crystal Displays

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NEW PROBLEMS

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Liquid crystal displays

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I. SCOPE

These problems illustrate a threshold property of liquid crystal displays (LCDs). Such displays are widely used in calculators, wristwatches, portable computers, electronic components, and various types of signs. By applying an electric field one can change the orientation of the liquid-crystal molecules and thus cause dramatic changes in the optical properties of the liquid-crystal material. The following problems show that when liquid-crystal material is confined between two surfaces as it is in an LCD, constraints due to the surfaces conspire with the electric field to produce a threshold phenomenon important for the design of displays. The problems also exhibit a striking analogy between the mathematics of the LCD threshold effect and the exact equation of motion of a pendulum.

The physics necessary to understand LCD behavior and the techniques needed to work out the details are learned in undergraduate courses in electromagnetism and mechanics. To work the following problems a student should be acquainted with simple vector algebra, the ideas of electric polarizability, electric susceptibility, the construction of a potential energy function, and either Euler's equation from the calculus of variations or an equivalent use of Lagrange's equations from classical mechanics. It is also necessary either to integrate a differential equation numerically or to evaluate elliptic integrals.

II. INTRODUCTION

Liquid crystals are fluids that behave like liquids and also show aspects of long-range order characteristic of crystals. The liquid-crystal phase of matter has molecules which maintain some degree of orientational order as they diffuse about in a fashion similar to liquids.[1–3] The molecules of a liquid-crystal phase are usually elongated, and it is their long axes that maintain a preferred direction. A snapshot of the molecules in a liquid-crystal phase is shown in Fig. 1 The direction of preferred orientation is called the "director," and is represented by the unit vector $\hat{n}$.

Because the molecules in a liquid crystal are not randomly oriented, a liquid-crystal phase can have properties which vary with direction. This is called "anisotropy," and liquid crystals are anisotropic fluids. For example, if the electric polarizability of a molecule parallel to its long axis is different from the polarizability across its long axis ($\alpha_\parallel$ and $\alpha_\perp$, respectively), then the electric susceptibility parallel to the director is different from the electric susceptibility perpendicular to the director ($\chi_\parallel$ and $\chi_\perp$, respectively). It is this difference in susceptibility, $\chi_a \equiv \chi_\parallel - \chi_\perp$, that causes a liquid crystal to orient in an electric field. If the anisotropy in the susceptibility is positive, i.e., $\chi_a > 0$, then the director will tend to align itself in the direction of the electric field. If $\chi_a < 0$, then the director will tend to align itself perpendicular to the electric field direction. The first of the problems below illustrates the energetics of this behavior in bulk material.

The situation is somewhat more complicated and interesting when the liquid crystal is placed between two slightly separated surfaces as shown in Fig. 2. Here a liquid crystal which tends to align parallel to an electric field lies between two glass plates a distance d apart. As a result of the treatment of the surfaces, liquid crystal molecules touching them become oriented parallel to them in what we adopt as the x direction. When an electric field is applied perpendicular to

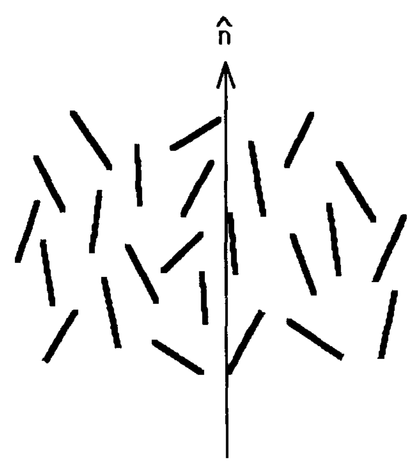

Fig. 1. Orientational order in a liquid crystal. $\hat{n}$ is the direction of preferred orientation of the long axes of the molecules and is called the director.

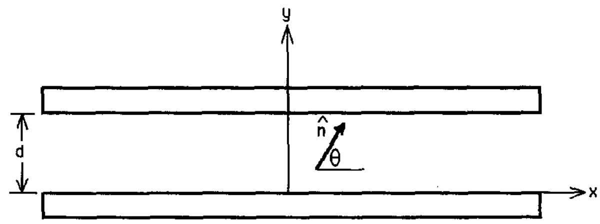

Fig. 2. The geometry of a liquid crystal display. The liquid crystal material is contained between the two pieces of glass. $\hat{n}$ represents the direction of orientational order in the liquid crystal and is a function of y only.

the surfaces, i.e., in the y direction, no alignment occurs until the field exceeds a threshold value. This threshold effect is illustrated by the last of the problems below.

III. PROBLEMS

A. Polarization of the liquid crystal

Note that if the electric field $\mathbf{E}$ is applied at some angle with respect to the director $\hat{n}$, there will be a component of the electric field $\mathbf{E}_\parallel$ parallel to $\hat{n}$ and a component $\mathbf{E}_\perp$ perpendicular to $\hat{n}$. Show that the polarization $\mathbf{P}=\epsilon_0(\chi_\parallel \mathbf{E}_\parallel + \chi_\perp \mathbf{E}_\perp)$ induced in the liquid crystal material can be written as

$$\mathbf{P}=\epsilon_0[\chi_\perp \mathbf{E}+\chi_a(\hat{n}\cdot\mathbf{E})\hat{n}]. \tag{1}$$

B. Electric potential energy of liquid crystals

Use the expression in Eq. (1) to find the potential energy per unit volume of the system. This can be done by calculating the work done by the electric field in turning the director from perpendicular to the field (taking that orientation to be $\theta=0$) to some angle θ. This is the integral with respect to θ of the torque per unit volume, $\mathbf{P}\times\mathbf{E}$. The electric potential energy density U_e, taken to be zero at $\theta=0$, will then be the negative of the work done by the field. Show that

$$U_e=-\tfrac{1}{2}\epsilon_0\chi_a E^2 \sin^2\theta. \tag{2}$$

C. Distortion energy produced by surfaces

If the proper surfactant is applied to the glass surfaces containing the liquid crystal, the director of the liquid crystal next to the glass is constrained to be parallel to the surfaces. Then if an electric field is applied perpendicular to the glass surfaces, the director in trying to align with the field will have a direction that varies through the fluid. Halfway between the glass surfaces the director will be aligned with the field as much as possible, but next to the glass surfaces it will be perpendicular to the field. This distortion increases the energy per unit volume of the liquid crystal just as compressing or expanding a spring from its equilibrium length increases the potential energy of the spring.

The effect of the surfaces is to make the x and y components of the director, i.e., n_x and n_y, vary as functions of y. In other words, dn_x/dy and dn_y/dy become nonzero; there is no variation of $\hat{n}$ in the x direction. These derivatives are measures of the amount of distortion introduced by applying an electric field to a liquid crystal that has the directions of its molecules anchored at the surfaces. If the distortion is not too large, then in analogy with a simple harmonic oscillator, the energy density increases with the square of the distortion, and we can write the energy density due to distortion as

$$U_d=\frac{1}{2}\,k\left(\frac{dn_x}{dy}\right)^2+\frac{1}{2}\,k\left(\frac{dn_y}{dy}\right)^2. \tag{3}$$

Since the two terms represent different types of distortion, there is no reason to expect that the coefficient k is the same for both. Nevertheless, the simplifying assumption that they are the same is a fairly good approximation in many cases.

Use the fact that $\hat{n}$ is a unit vector to show that the total energy density, U_d+U_e, can be expressed as

$$U=\frac{1}{2}\,k\left(\frac{d\theta}{dy}\right)^2-\frac{1}{2}\,\epsilon_0\chi_a E^2\sin^2\theta. \tag{4}$$

D. Distribution of the director

The next problem is to find the director configuration which minimizes the sum of the electric and distortion energies.

Physically the director will distribute itself to minimize the total energy, which is proportional to the integral $\int_0^d U\,dy$ where d is the spacing between the two glass surfaces. In other words we want to know what function $\theta(y)$ will yield the minimum value for this integral.

This is an extremum problem that you can solve by invoking the calculus of variations and using Euler's equation. Equivalently, you can look upon Eq. (4) as a "kinetic energy" term plus a "potential energy" term with y playing the role of the time variable. Then finding the function $\theta(y)$ which minimizes the total energy is analogous to finding the motion that minimizes the Lagrangian for the situation.[4]

(a) Show that $\theta(y)$ must be a solution of

$$\xi_d^2\frac{d^2\theta}{dy^2}+\sin\theta\cos\theta=0, \tag{5}$$

where ξ_d is defined to be ξ/d where ξ, which is an important characteristic length of the physical system, is defined by the expression

$$\xi^2=\frac{k}{\epsilon_0\chi_a E^2}. \tag{6}$$

(b) Now solve the differential equation for the function $\theta(y)$. You can do this numerically or in terms of elliptic integrals.

(c) When you do this there is a surprise! For ξ_d greater than approximately 0.32 there is no solution except $\theta(y)=0$. This means that below some value of the electric field, no distortion whatsoever occurs. To exhibit this effect calculate the value of θ_m at the midpoint between the plates for different strengths of electric field, and plot θ_m as a function of electric field strength.

(d) Find an analytic expression for the threshold electric field E_t. Then calculate the minimum voltage that must be applied to a typical LCD which has $d=10$ μm and a liquid crystal with $k=10^{-11}$ N and $\chi_a=11$ in order to produce any alignment of the director along the direction of the electric field.

IV. SOLUTIONS

A. Polarization of the liquid crystal

The problem here is to express the polarization in terms of the complete field $\mathbf{E}$ rather than its components $\mathbf{E}_\parallel$ and $\mathbf{E}_\perp$.

The magnitude of $\mathbf{E}_\parallel$ is the projection of $\mathbf{E}$ onto the director $\hat{n}$ which is just $\mathbf{E}\cdot\hat{n}$, and because its direction is $\hat{n}$ you can write

$$\mathbf{E}_\parallel=(\mathbf{E}\cdot\hat{n})\hat{n}.$$

Then, because $\mathbf{E}_\perp=\mathbf{E}-\mathbf{E}_\parallel$ it follows that

$$\mathbf{E}_\perp=\mathbf{E}-(\mathbf{E}\cdot\hat{n})\hat{n}.$$

Substituting these into the expression $\epsilon_0(\chi_\parallel\mathbf{E}_\parallel+\chi_\perp\mathbf{E}_\perp)$ gives

$$\begin{aligned}\mathbf{P}&=\epsilon_0[\chi_\parallel(\mathbf{E}\cdot\hat{n})\hat{n}+\chi_\perp\mathbf{E}_\perp]\\&=\epsilon_0[\chi_\parallel(\mathbf{E}\cdot\hat{n})\hat{n}+\chi_\perp(\mathbf{E}-(\mathbf{E}\cdot\hat{n})\hat{n})]\\&=\epsilon_0[\chi_\perp\mathbf{E}+(\chi_\parallel-\chi_\perp)(\mathbf{E}\cdot\hat{n})\hat{n}]\\&=\epsilon_0[\chi_\perp\mathbf{E}+\chi_a(\mathbf{E}\cdot\hat{n})\hat{n}],\end{aligned}$$

where in the last step we have used the definition $\chi_a\equiv\chi_\parallel-\chi_\perp$.

B. Electric potential energy of liquid crystals

Because the torque per unit volume is just $\mathbf{P}\times\mathbf{E}$, the energy W stored per unit volume in the liquid when the director is rotated through an angle θ from perpendicular to $\mathbf{E}$ will be

$$\begin{aligned}W&=\int_0^\theta|\mathbf{P}\times\mathbf{E}|d\theta'\\&=\epsilon_0\chi_a\int_0^\theta(\hat{n}\cdot\mathbf{E})|\hat{n}\times\mathbf{E}|d\theta'\\&=\epsilon_0\chi_aE^2\int_0^\theta\sin\theta'\cos\theta'\,d\theta',\end{aligned}$$

$$W=\frac{1}{2}\epsilon_0\chi_aE^2\sin^2\theta.$$

If we define the zero of the potential energy density to be at $\theta=0$, then the potential energy density as a function of θ is just the negative of W the work done by the field

$$U_e=-\tfrac{1}{2}\epsilon_0\chi_aE^2\sin^2\theta. \tag{7}$$

C. Distortion energy produced by surfaces

To express Eq. (3) in terms of the angle θ make use of the fact that $\hat{n}$ is a unit vector so that

$$n_x=\cos\theta,$$

$$n_y=\sin\theta,$$

$$\frac{dn_x}{dy}=-\sin\theta\frac{d\theta}{dy},$$

$$\frac{dn_y}{dy}=\cos\theta\frac{d\theta}{dy}.$$

Substituting these expressions into Eq. (3) gives

$$U_d=\frac{1}{2}k\left[\sin^2\theta\left(\frac{d\theta}{dy}\right)^2+\cos^2\theta\left(\frac{d\theta}{dy}\right)^2\right]=\frac{1}{2}k\left(\frac{d\theta}{dy}\right)^2.$$

Therefore, the total energy density can be expressed as

$$U=U_d+U_e=\frac{1}{2}k\left(\frac{d\theta}{dy}\right)^2-\frac{1}{2}\epsilon_0\chi_aE^2\sin^2\theta, \tag{8}$$

which is the same as Eq. (4).

D. Distribution of the director

(a) The total energy stored in the liquid crystal is the volume integral of U taken over the entire volume of the fluid. The director will distribute its orientation in such a way as to minimize this total energy. Because U depends only on y, the integral will be proportional to $\int U\,dy$, and the problem reduces to finding the function $\theta(y)$ that minimizes this one-dimensional integral. Euler showed that the function U that minimizes an integral $\int_0^\theta U(\theta,d\theta/dy,y)dy$ must be a solution to the differential equation

$$\frac{\partial U}{\partial\theta}-\frac{d}{dy}\frac{\partial U}{\partial(d\theta/dy)}=0.$$

Substituting Eq. (8) into Euler's equation gives

$$k\frac{d^2\theta}{dy^2}+\epsilon_0\chi_aE^2\sin\theta\cos\theta=0. \tag{9}$$

To express Eq. (9) in terms of a dimensionless quantity, make a change of variable from y to y/d and then replace y/d with the symbol y which will then be a length measured in units of d. This will yield

$$\frac{k}{d^2}\frac{d^2\theta}{dy^2}+\epsilon_0\chi_aE^2\sin\theta\cos\theta=0.$$

Then let $\xi^2=k/(\epsilon_0\chi_aE^2)$ and rewrite the previous equation as

$$\left(\frac{\xi}{d}\right)^2\frac{d^2\theta}{dy^2}+\sin\theta\cos\theta=0.$$

It is convenient to express this in terms of a dimensionless parameter $\xi_d=\xi/d$. The equation you need to solve then takes the form

$$\xi_d^2\frac{d^2\theta}{dy^2}+\sin\theta\cos\theta=0. \tag{10}$$

Notice that this equation can also be written as

$$\xi_d^2\frac{d^2(2\theta)}{dy^2}+\sin2\theta=0,$$

which is the differential equation of motion for a large-amplitude pendulum if the angle θ is half the angle from the downward vertical to the pendulum axis. The dimensionless parameters of Eq. (10) have the advantage that you can solve the equation once and apply it to all cases.

(b) In spite of how simple this equation appears, there is no solution $\theta(y)$ that can be expressed in terms of familiar functions. You can obtain solutions in terms of elliptic integrals or by numerical integration of Eq. (10). In this latter case, you would choose a value for ξ_d (remembering that ξ_d is inversely proportional to the applied electric field), guess a value of $d\theta/dy$ at $y=0$, and recall that the anchoring at the surface makes $\theta=0$ at $y=0$ and $y=1$. Then integrate numerically from $y=0$ to $y=1$ to obtain a function $\theta(y)$. In general this function will not return to 0 at $y=1$ as it should. However, by seeing whether the trial value of $d\theta/dy$ at $y=0$ is too low or too high, you can quickly zero in on the proper value. When $\theta(y)$ returns to 0 at $y=1$, you will have a solution for your chosen value of ξ_d. Repeating the procedure for other values of ξ_d will produce solutions for other values of the electric field.

To solve Eq. (10) in terms of elliptic integrals, find the first integral of Eq. (10) by multiplying by $d\theta/dy$ and integrating.

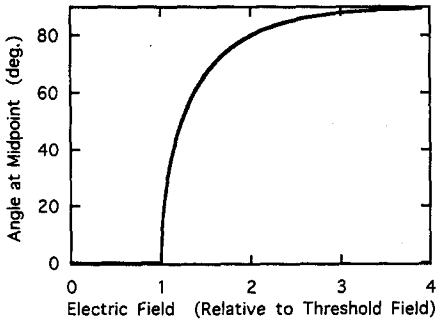

Fig. 3. The dependence on electric field strength of θ_m, the angle the director makes with the x axis at a point midway between the two pieces of glass.

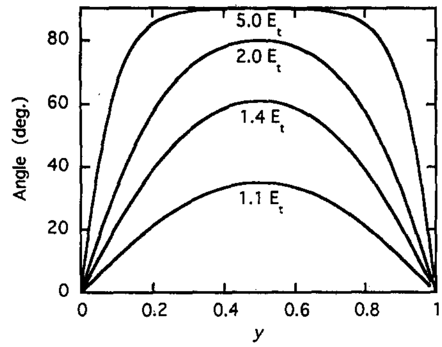

Fig. 4. The variation of θ, the angle the director makes with the x axis, as a function of $y=y/d$ for different electric field strengths. E_t is the threshold electric field needed for any distortion to occur.

The integration constant can be written in terms of θ_m, the value of θ at $y=1/2$, by noting that $d\theta/dy=0$ at θ_m. This gives

$$\frac{d\theta}{dy}=\frac{1}{\xi_d}\sqrt{\sin^2\theta_m-\sin^2\theta}. \tag{11}$$

Although Eq. (11) cannot be integrated in closed form, it can be expressed as an elliptic integral. This is done by introducing a new parameter $m=\sin^2\theta_m$ and making the following change of variable

$$t=\frac{\sin\theta}{\sin\theta_m},$$

to obtain

$$\int_0^1\frac{dt}{\sqrt{1-t^2}\sqrt{1-mt^2}}\equiv K(m).$$

$K(m)$ is known as the complete elliptic integral of the first kind.[5] For our problem, the y integration is from $y=0$ to $y=1/2$ so that

$$K(m)=\frac{1}{2\xi_d}.$$

An easy way to generate a graph of θ_m versus E/E_t is to pick a value for θ_m, then calculate $m=\sin^2\theta_m$ and finally use tables or software to find

$$\frac{E}{E_t}=\frac{1}{\xi_d\pi}=\frac{2}{\pi}K(m).$$

The result of this procedure or of the equivalent numerical integration is shown in Fig. 3.

You can also find the variation of θ with y for a particular value of E/E_t in terms of elliptic integrals. Instead of integrating from $y=0$ to $y=1/2$, this time integrate from $y=0$ to some arbitrary value of y, with θ going from 0 to the value of θ corresponding to this value of y. After using the substitution $1/\xi_d=2K(m)$, the result is

$$\int_0^{\sin\phi}\frac{dt}{\sqrt{1-t^2}\sqrt{1-mt^2}}=2K(m)y,$$

where $\sin\phi\equiv\sin\theta/\sin\theta_m$. This integral is an elliptic integral of the first kind, $F(\phi|m)$,[5] and the equation can be written as

$$y=\frac{F(\phi|m)}{2K(m)}. \tag{12}$$

To obtain a graph of θ versus y, use the procedure described above to find the value of $K(m)$ given a value of θ_m, and then pick values for θ less than θ_m. For each value of θ calculate

$$\phi=\sin^{-1}\left(\frac{\sin\theta}{\sin\theta_m}\right)$$

and use tables or software to find the corresponding value of y from Eq. (12). Using various values of the electric field and either this procedure or numerical integration, you will get the results shown in Fig. 4.

(c) The existence of a threshold is apparent when solving the equation numerically, since the only solution possible for $1/\xi_d<\pi$ is that of $\theta=0$ everywhere.

The reason for the existence of a threshold field can be understood from Eq. (8). The first term is minimized when θ has a constant value, while the second term is minimized for $\theta=90°$. When the electric field is below the threshold value, the decrease in energy because the director is more parallel to the electric field is not as great as the increase in energy due to the distortion necessary to change the orientation of the director. Hence, no distortion occurs.

To find an expression for the threshold electric field consider Eq. (11). You can integrate this equation when θ and $\theta_m\ll 1$, i.e., the situation where the director has just begun to twist. Then $\sin\theta\approx\theta$ and $\sin\theta_m\approx\theta_m$ and

$$\int_0^{\theta_m}\frac{d\theta}{\sqrt{\theta_m^2-\theta^2}}=\int_0^{1/2}\frac{dy}{\xi_d},$$

which gives

$$\frac{1}{2\xi_d}=\frac{\pi}{2}. \tag{13}$$

Consequently, the electric field must equal at least

$$E_t=\frac{\pi}{d}\sqrt{\frac{k}{\epsilon_0\chi_a}} \tag{14}$$

before distortion can occur.

You can also find Eq. (14) by using tables to examine the behavior of the elliptic integral $K(m)$. The tables will show you that $K(m)$ has a minimum value of $\pi/2$ at $m=0$ and becomes infinite as m approaches 1. This consideration yields Eq. (14) once again.

Equation (14) shows that, as you would expect, the smaller the distance between the surfaces, the greater the electric field necessary to distort the liquid crystal. Likewise, the threshold field goes up when the "spring constant" k increases, and it goes down when the electric susceptibility anisotropy χ_a is larger. Both of these dependences make sense.

The thickness of a typical LCD is about 10 μm. If a liquid crystal with $k=10^{-11}$ N and $\chi_a=11$ is used, then the voltage, $E_t d$, needed to produce the threshold field is about 1 V—a very convenient value. In practice, a voltage several times larger than this is used in order to align the liquid crystal director along the field direction throughout a large portion of the cell.

V. REMARKS

The situation is a little more complicated in the most common type of liquid crystal display, the twisted nematic LCD. The zero field director configuration is one in which the director is always parallel to the glass surfaces but twists by 90° in going from one surface to the other. This configuration rotates the plane of polarization of light by 90° as it passes through the liquid crystal, thus allowing the light to get through the crossed polarizers on the outside of each piece of glass. When an electric field above the threshold is applied, the director is parallel to the field throughout most of the cell and only twists in thin boundary regions next to the glass. Consequently, the plane of polarization of light passing through this director configuration is rotated very little, and the cell appears black due to the crossed polarizers. This type of LCD is bright with no applied field and dark with an applied field.

Because of the twist, the energetics are different and the expression for the threshold electric field is more complicated. More realistic calculations also include an interaction between the liquid crystal and the surfaces rather than simply applying the boundary conditions as we did.

The electric field is applied by transparent electrodes on the inside surfaces of the glass. These electrodes are made by coating the glass with a thin layer of indium-tin oxide. The film of surfactant needed to orient the liquid crystal is then applied on top of the indium-tin oxide.

[1]P. J. Collings, *Liquid Crystals: Nature's Delicate Phase of Matter* (Princeton University, Princeton, 1990).

[2]P. G. deGennes and J. Prost, *The Physics of Liquid Crystals*, 2nd ed. (Clarendon, Oxford, 1993).

[3]S. Chandresekhar, *Liquid Crystals*, 2nd ed. (Cambridge University, Cambridge, 1992).

[4]G. R. Fowles, *Analytical Mechanics*, 2nd ed. (Prentice-Hall, Englewood Cliffs, NJ, 1989).

[5]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965).

TESTING PARITY CONSERVATION

Why was it so that among the multitude of experiments on β-decay, the most exhaustively studied of all the weak interactions, there was no information on the conservation of parity in the weak interactions? The answer derives from a combination of two reasons. First, the fact that the neutrino does not have a measurable mass introduces an ambiguity that rules out indirect information on parity conservation from such simple experiments as the spectrum of β-decay. Second, to study directly parity conservation in β-decay it is not enough to discuss nuclear parities, as one had always done. One must study parity conservation of the *whole* decay process. In other words, one must design an experiment that tests right-left symmetry in the decay. Such experiments were not done before.

Once these points were understood it was easy to point out what were the experiments that would unambiguously test the previously untested assumption of parity conservation in the weak interactions. Dr. Lee and I proposed in the summer of 1956 a number of these tests concerning β-decay, π-μ, μ-e and strange-particle decays. The basic principles involved in these experiments are all the same: *One constructs two sets of experimental arrangements which are mirror images of each other, and which contain weak interactions. One then examines whether the two arrangements always give the same results in terms of the readings of their meters* (or counters). If the results are not the same, one would have an unequivocal proof that right-left symmetry, as we usually understand it, breaks down.

Chen Ning Yang, "The law of parity conservation and other symmetry laws of physics," (Nobel Lecture, December 11, 1957, reprinted in *Nobel Lectures, Physics*, Vol. 3, 1942–1962, Elsevier Amsterdam, 1964).