Thin-Shell Mixing In Radiative Wind-Shocks And The L-x Similar To L-bol Scaling Of O-Star X-Rays

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Thin-shell mixing in radiative wind-shocks and the $L_x \sim L_{bol}$ scaling of O-star X-rays

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ABSTRACT
X-ray satellites since Einstein have empirically established that the X-ray luminosity from single O-stars scales linearly with bolometric luminosity, $L_x \sim 10^{-7}L_{bol}$. But straightforward forms of the most favoured model, in which X-rays arise from instability-generated shocks embedded in the stellar wind, predict a steeper scaling, either with mass-loss rate $L_x \sim M \sim L_{bol}^{1.7}$ if the shocks are radiative or with $L_x \sim M^2 \sim L_{bol}^{3.4}$ if they are adiabatic. This paper presents a generalized formalism that bridges these radiative versus adiabatic limits in terms of the ratio of the shock cooling length to the local radius. Noting that the thin-shell instability of radiative shocks should lead to extensive mixing of hot and cool material, we propose that the associated softening and weakening of the X-ray emission can be parametrized as scaling with the cooling length ratio raised to a power $m$, the ‘mixing exponent’. For physically reasonable values $m \approx 0.4$, this leads to an X-ray luminosity $L_x \sim M^{1.6} \sim L_{bol}$ that matches the empirical scaling. To fit observed X-ray line profiles, we find that such radiative-shock-mixing models require the number of shocks to drop sharply above the initial shock onset radius. This in turn implies that the X-ray luminosity should saturate and even decrease for optically thick winds with very high mass-loss rates. In the opposite limit of adiabatic shocks in low-density winds (e.g. from B-stars), the X-ray luminosity should drop steeply with $M^2$. Future numerical simulation studies will be needed to test the general thin-shell mixing ansatz for X-ray emission.

Key words: shock waves — stars: early-type — stars: mass-loss — stars: winds, outflows — X-rays: stars.

1 INTRODUCTION
Since the 1970s X-ray satellite missions like Einstein, ROSAT, and most recently Chandra and XMM–Newton have found hot, luminous, O-type stars to be sources of soft ($\leq 1$ keV) X-rays, with a roughly linear scaling between the X-ray luminosity and the stellar bolometric luminosity, $L_x \sim 10^{-7}L_{bol}$ (Chlebowski, Harnden & Sciortino 1989; Kudritzki et al. 1996; Berghoefer et al. 1997; Sana et al. 2006; Güdel & Nazé 2009; Nazé 2009; Nazé et al. 2011). In some systems with harder (a few keV) spectra and/or higher $L_x$, the observed X-rays have been associated with shock emission in colliding wind binary (CWB) systems (Stevens, Blondin & Pollock 1992; Gagné et al. 2012), or with magnetically confined wind shocks (Babel & Montmerle 1997; Wade 2012). But in putatively single, non-magnetic O-stars, the most favoured model is that the X-rays are emitted from embedded wind shocks that form from the strong, intrinsic instability (the ‘line-deshadowing instability or LDI) associated with the driving of these winds by line scattering of the star’s radiative flux (Owocki, Castor & Rybicki 1988; Feldmeier, Puls & Pauldrach 1997b; Dessart & Owocki 2003; Sundqvist & Owocki 2012).

This LDI can be simply viewed as causing some small ($\lesssim 10^{-3}$) fraction of the wind material to pass through an X-ray emitting shock, implying in the case that the full shock energy is suddenly radiated away such that the X-ray luminosity should scale with the wind mass-loss rate, $L_x \sim M$. But within the standard Castor, Abbott & Klein (1975, hereafter CAK) model for such radiatively driven stellar winds, this mass-loss rate increases with luminosity2 as $M \sim L_{bol}^{0.5} \sim L_{bol}^{1.7}$, where the latter scaling uses a typical CAK power index $\alpha \approx 0.6$ (Puls, Springmann & Lennon 2000). This then implies a superlinear scaling for X-ray to bolometric luminosity, $L_x \sim L_{bol}^{3.4}$.

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1 i.e. extending over 2 dex in $L_{bol}$, with a typical scatter of $\sim \pm 0.5$ dex

2 For simplicity, this ignores a secondary scaling of luminosity with mass; see Section 3.4.
$L_x \sim L_{bol}^{0.7}$, which is too steep to match the observed, roughly linear law.

In fact, the above sudden-emission scaling effectively assumes that the shocks are radiative, with a cooling length that is much smaller than the local radius, $\ell \ll r$. In the opposite limit $\ell \gg r$, applicable to lower density winds for which shocks cool by adiabatic expansion, the shock emission scales with the X-ray emission measure, $EM \sim \int \rho^2 dV$, leading then to an even steeper scaling of X-ray versus bolometric luminosity, $L_x \sim M^2 \sim L_{bol}^{1.4}$.

Both these scalings ignore the effect of bound-free absorption of X-rays by the cool, unshocked material that represents the bulk of the stellar wind. Owocki & Cohen (1999, hereafter OC99) showed that accounting for wind absorption can lead to an observed X-ray luminosity that scales linearly with $L_{bol}$, but this requires specifying ad hoc a fixed radial decline in the volume filling factor for X-ray emitting gas. We show below (Section 2.4) that this filling factor should actually be strongly affected by the level of radiative cooling. Moreover, while modern observations of spectrally resolved X-ray emission profiles by Chandra and XMM–Newton do indeed show the expected broadening from shocks embedded in the expanding stellar wind, the relatively weak blue–red asymmetry indicates that absorption effects are modest in even the densest OB-star winds (Cohen et al. 2010, 2011). Since many stars following the $L_x \sim L_{bol}$ empirical law have weaker winds that are largely optically thin to X-rays (Cohen, Cassinelli & Macfarlane 1997; Nazé et al. 2011), it now seems clear that absorption cannot explain this broad $L_x$ scaling.

The analysis here examines instead the role of radiative cooling, and associated thin-shell instabilities (Vishniac 1994; Walder & Folini 1998; Schure et al. 2009; Parkin & Pittard 2010), in mixing shock-heated material with cooler gas, and thereby reducing and softening the overall X-ray emission. As summarized in Fig. 1, for a simple parametrization that this mixing reduction scales with a power (the ‘mixing exponent’ $m$) of the cooling length, $\ell^m$, we find that the linear $L_x \sim L_{bol}$ law can be reproduced by assuming $m \approx 0.4$. The development below quantifies and extends a preliminary conference presentation of this thin-shell mixing ansatz (Owocki et al. 2012).

Specifically, to provide a quantitative basis for bridging the transition between radiative and adiabatic shock cooling, Section 2 first analyses in detail the X-ray emission from a simplified model of a single, standing shock in steady, spherically expanding outflow. Section 3 then generalizes the resulting simple bridging law to account for thin-shell mixing, and applies this in a simple exospheric model to derive general scalings for X-ray luminosity from a wind with multiple, instability-generated shocks assumed to have a power-law number distribution in wind radius. A further application to computation of X-ray line profiles (Section 4) provides constraints on the mixing and shock-number exponents needed to match observed X-ray emission lines. Following a brief presentation (Section 5) of full integral solutions for X-ray luminosity to complement the general scaling laws in Section 3, Section 6 concludes with a brief summary and outlook for future work.

### 2 Standing-Shock Model

#### 2.1 Energy Balance

Most previous analyses of X-rays from massive stars (e.g. Wojdowski & Schulz 2005; Cohen et al. 2011) have been cast in terms of a density-squared emission measure from some fixed volume filling factor for X-ray emitting gas (see however Krolik & Raymond 1985; Hillier et al. 1993; Feldmeier et al. 1997a; Antokhin, Owocki & Brown 2004). So let us begin by showing

![Figure 1. Summary sketch of the key concepts and results of the scaling analysis in this paper. The illustration of the cooling zone from a wind shock shows associated scalings for X-ray luminosity $L_x$ with mass-loss rate $\dot{M}$ and bolometric luminosity $L_{bol}$, based on the conversion factor $k_x$ of wind kinetic energy into X-rays, which for radiative shocks is a constant, but for adiabatic shocks is reduced by the ratio of the radius to cooling length, $r/\ell < 1$. (See equation 12 in the text.) In addition, thin-shell mixing of such radiative shocks is then postulated to lead to a reduction of the X-ray emitting fraction that scales as a power law of the cooling length, $k_x \sim \ell^m$. For CAK wind index $\alpha$, a mixing exponent $m = 1 - \alpha$ leads to the observationally inferred linear scaling of X-rays with bolometric luminosity, $L_x \sim L_{bol}$.](image-url)
explicitly how that picture must be modified to account for the
effects of radiative cooling, which can be important and even domi-
nant for O-star wind shocks. By focusing on the simple example of
a steady, standing wind-shock, it is possible to carry out an analytic
analysis that derives, more or less from first principles, a simple
bridging law between the scalings for adiabatic versus radiative
shocks.

Specifically, let us consider an idealized model in which a
spatially symmetric, steady-state stellar wind with mass-loss rate \( \dot{M} \)
and constant, highly supersonic flow speed \( V_\infty \) undergoes a strong,
standing shock at some fixed radius \( r = r_s \). Relative to an unshocked
wind with density \( \rho_w = \dot{M}/4\pi r V_\infty^2 \), the post-shock flow at \( r > r_s \)
has a density that is a factor \( \rho / \rho_w = 4 \) higher, with a post-shock
speed that is a factor \( v / V_\infty = 1/4 \) lower. The reduction in flow ki-
netic energy results in a high, immediate post-shock temperature,
\[
T_s = \frac{3}{16} \frac{\mu V_\infty^2}{k} = 14 \text{ MK} \left( \frac{V_\infty}{1000 \text{ km s}^{-1}} \right)^2 ,
\]
where \( k \) is Boltzmann’s constant and the latter evaluation assumes
a standard molecular weight \( \mu = 0.62 m_p \), with \( m_p \) the proton mass.

But following this sudden shock increase, the combined effects
of adiabatic expansion and radiative cooling cause the tem-
perature \( T \) to decrease outward. For pressure \( P = \rho k T / \mu \) and internal
energy density \( \epsilon = (3/2) \rho \), the steady-state energy balance for a
general vector velocity \( v \)
is
\[
\nabla \cdot (\rho \epsilon v) = - P \nabla \cdot v - \rho^2 \Lambda_m(T) ,
\]
where \( \Lambda_m \equiv \Lambda/\mu_\mu_p \), with \( \Lambda(T) \) the optically thin cooling func-
tion (e.g. Smith et al. 2001), and \( \mu_\mu \) and \( \mu_p \), respectively, the mean
mass per electron and per proton.

In general, we should also include a detailed momentum equation
to account for possible acceleration of the post-shock flow, for
example from the inward pull of stellar gravity, or the outward push
of the gas pressure gradient. But the analysis here is greatly simplified
if we make the reasonable assumption that these two counteract-
ing accelerations roughly cancel, and so give a constant speed \( v = V_\infty /4 \)
for all \( r > r_s \). For this case of a steady, spherical, constant-
speed post-shock outflow, the vector velocity equation (2) reduces to
a simple differential equation for the decline in temperature with
radius \( r \),
\[
\frac{dT}{dr} = - \frac{4}{3} \frac{T}{r} - \frac{2 \mu}{3 k} \rho \Lambda_m(T) ,
\]
wherein the first and second terms on the right-hand-side, respec-
tively, represent the effects of adiabatic expansion and radiative
cooling. This can alternatively be cast in terms of a temperature
scalelength,
\[
\frac{1}{H_T} = - \frac{1}{T} \frac{dT}{dr} = \frac{4}{3r} + \frac{\kappa_p \rho}{3} ,
\]
where
\[
\kappa_p(T) \equiv 8 \mu \Lambda_m(T) / kT V_\infty
\]
is a mass cooling coefficient (with CGS units \( \text{cm}^{-2} \text{g}^{-1} \)), defined as
the inverse of a characteristic cooling column mass. The corre-
sponding cooling length is given by \( \ell = 4/\kappa_p \rho = 1/\kappa_p \rho_w \), defined
such that the radiative and adiabatic cooling terms are equal\(^4\) when
\( \ell = r \).

### 2.2 X-ray luminosity

For any local post-shock temperature \( T \), let \( f_s(T) \) represent the frac-
tion of radiation emitted in an X-ray bandpass of interest. Neglect-
ing for now any wind absorption, the total X-ray luminosity from
this single standing shock is then given by radial integration of the
associated X-ray emission,
\[
L_{\text{v}} = 4\pi \int_{r_s}^{\infty} \rho^2 \Lambda_m(T) f_s(T) r^2 d r .
\]
Using (4), this can be recast as an integral over temperature,
\[
L_{\text{v}} = 12\pi \int_0^{T_s} r_s^3 \frac{\rho^2 \Lambda_m(T)}{4 + \kappa_p(T) \rho r_s} f_s(T) \frac{dT}{T} .
\]
with the latter approximation\(^3\) uses single-point trapezoidal inte-
gration, assuming that \( f_s \) declines from its post-shock value \( f_s(T_s) \) to
zero over a temperature range \( \delta T_s \) from the initial post-shock tem-
perature \( T_s = T(r_s) \). Here \( \rho_w = \dot{M} / (4\pi V_\infty r_s^2) \) is the wind density
just before the shock, and
\[
\kappa_p \equiv \kappa_p(T_s) = \frac{128 \Lambda_m(T_s)}{3 V_\infty^2}
\]
\[
= 2\sqrt{3} \Lambda_m(T_s) \left( \frac{kT_s}{\mu} \right)^{-3/2}
\]
\[
\approx 1000 \text{ cm}^2 \text{ g}^{-1} T_s^{-2/3} \approx 750 \text{ cm}^2 \text{ g}^{-1} T_{\text{kev}}^{-2} ,
\]
with \( T_s \equiv T / 10^7 \) K and \( T_{\text{kev}} = kT_s / \text{keV} \). The numerical evalua-
tion in (8c) assumes an approximate fit to the cooling function,
\( \Lambda(T) \approx 4.4 \times 10^{-25} / T_{\text{kev}}^2 \text{ erg cm}^{-3} \text{ s}^{-1} \), over the rele-
vant range of shock temperatures, \( 10^{6.5} K < T_s < 10^{7.5} K \) (Schure et al. 2009).
Recall that the shock cooling length is set by \( \ell_s = 1/\kappa_p \rho_w \).

For context, the mass absorption coefficient for bound-free ab-
sorption of X-rays when smoothed over bound-free edges, also
roughly follows an inverse-square scaling with energy. Over the
relevant energy range 0.5–2 keV, we can use the the opacity curva-
tures of, e.g. Cohen et al. (2010), Leutenegger et al. (2010) or Hervé et al.
(2012) to write an approximate scaling form,
\[
\kappa_{\text{bd}} \approx 30 \text{ cm}^2 \text{ g}^{-1} E_{\text{kev}}^{-2} ,
\]
where \( E_{\text{kev}} \) is now the X-ray photon energy in keV.

By casting the cooling strength in an opacity form normally used
to describe absorption, we are thus able to make direct comparisons

\(^3\) This was actually noted explicitly by Zhukov & Palla (2007), but their
results were nonetheless still cast in terms of a density-squared emission
measure that is not appropriate for radiative shocks.

\(^4\) The ratio \( \ell / r \) differs only by an order-unity factor from the ratio of cooling
to escape time, \( x \equiv \ell_{\text{cool}} / \ell_{\text{esc}} \), defined by Stevens et al. (1992) to charac-
terize the transition from radiative to adiabatic shocks in colliding stellar winds.

\(^5\) Aside from \( f_s(T) \), the combination of other factors in the integrand for
\( T > r_s \) becomes constant in \( T \) in the radiative limit \( \kappa_p \rho r \gg 1 \), and scales as \( T^{-3/4} \) in the opposite, adiabatic limit. For \( f_s(T) \) that declines roughly linearly
with temperature, the approximate trapezoidal integration (7b) thus becomes
nearly exact in the radiative limit, while mildly underestimating the actual
value in the adiabatic limit, for example by about 15 per cent for \( \delta T_s = T_s / 2 \).
between cooling and absorption, and so characterize their respective domains of importance. This is further facilitated by the quite fortunate coincidence that both have similar inverse-square scalings with their associated energy.

Since these respective energies are usually roughly comparable, \( E_{\text{rad}, s} \approx E_{\text{adi}, s} \), the fact that the numerical factor for \( \kappa_{\text{ss}} \) is about 25 times greater than for \( \kappa_{\text{ad}} \) means that cooling can become important even in winds that are too low density to have significant absorption. As such, winds with adiabatic shocks are always optically thin, whereas shocks in optically thick winds are always radioactive. Moreover, as detailed in Section 3, even in the bulk of O-star winds for which X-ray absorption is weak or marginal, the structure of X-ray emission associated wind shocks should be dominated by radiative cooling.

### 2.3 Bridging law between radiative and adiabatic limits

Noting that the pre-shock kinetic energy luminosity of the wind is

\[
L_w = 2\pi r_s^2 \rho_{\infty} V_{\infty}^2 = MV_{\infty}^2/2, \tag{10}
\]

can we use (8a) to eliminate the cooling function \( \Lambda_m \) from (7b), recasting the X-ray luminosity scaling as a ‘bridging law’ between the radiative and adiabatic shock limits,

\[
L_{\text{ss}} = f_{\text{ss}} \frac{9}{16} \frac{L_w}{1 + \ell_s/r_s}, \tag{11}
\]

where \( f_{\text{ss}} = f_s(T_s) \delta_{T_s}/2T_s \) is now a cooling-integrated shock X-ray fraction.

Note here that the combination of factors multiplying \( L_w \) on the right-hand-side of (11) is just the kinetic energy conversion factor introduced in the summary Fig. 1,

\[
k_s = \frac{9}{16} \frac{f_{\text{ss}}}{1 + \ell_s/r_s}. \tag{12}
\]

For high-density, radiative shocks with \( \ell_s \ll r_s \),

\[
L_{\text{ss, rad}} = f_{\text{ss}} \frac{9}{16} L_w = f_{\text{ss}} \frac{9}{32} MV_{\infty}^2. \tag{13}
\]

As a physical interpretation, 9/16 is just the fraction of wind kinetic energy that is converted to post-shock heat, which is radiated away before any losses to adiabatic expansion, with the fraction \( f_{ss} \) emitted in the X-ray bandpass of interest. Note that this scales linearly with density and thus mass-loss rate, showing that a standard density-squared emission measure does not represent an appropriate scaling for emission from radiative shocks.

For lower density, adiabatic shocks with \( \ell_s \gg r_s \), this X-ray emission is reduced by the ratio \( r_s/\ell_s \), giving the scalings,

\[
L_{\text{ss, ad}} = f_{\text{ss}} \frac{9}{16} \frac{r_s}{\ell_s} L_w \tag{14a}
\]

\[
= f_{\text{ss}} \frac{9\pi}{8} r_s^3 V_{\infty}^3 \kappa_{\text{ss}} \rho_{\infty}^2 \tag{14b}
\]

\[
= f_{\text{ss}} \frac{48\pi r_s^3 \Lambda_m(T_s) \rho_{\infty}^2}{\ell_s} \tag{14c}
\]

\[
= f_{\text{ss}} \frac{32 \Lambda_m(T_s) \rho_{\infty}^2}{\pi r_s} \left( \frac{M}{\infty} \right)^2, \tag{14d}
\]

which thus recovers the density-squared scaling for X-ray luminosity, showing that emission measure does provide an appropriate scaling for emission from adiabatic shocks.

Finally, note that the general bridging law (11) can alternatively be written as a modification of either the radiative or adiabatic scaling,

\[
L_{\text{ss}} = \frac{L_{\text{ss, rad}}}{1 + \ell_s/r_s} \tag{15a}
\]

\[
= \frac{L_{\text{ss, ad}}}{1 + r_s/\ell_s}. \tag{15b}
\]

This shows that the X-ray luminosity is always limited to be somewhat below the smaller of the radiative or adiabatic luminosities, i.e. \( L_{\text{ss}} \lesssim \min(L_{\text{ss, rad}}, L_{\text{ss, ad}}) \).

### 2.4 Local X-ray emissivity

To facilitate application of these single standing-shock scalings to the more complex case of multiple embedded wind shocks generated from the LDI, let us next recast these results in terms of the local X-ray emission from an individual shock. Since X-ray emission arises from collision of ions and electrons, it is common to write the X-ray emissivity (per unit volume and solid angle) as scaling with the square of the local density,

\[
\eta_s = C_s f_s \rho^2, \tag{16}
\]

where \( C_s \) is a constant that depends on the shock strength and atomic physics, and \( f_s \) represents a local volume filling factor for shocked gas that is sufficiently hot to emit X-rays. If each individual shock has an associated filling factor \( f_s \) proportional to its post-shock cooling length, then the total filling factor from an ensemble of shocks can be written as

\[
f_s(r) = f_{s \text{ss}} \frac{dN_s}{d\ln r} = f_{s \text{ss}} n_s(r), \tag{17}
\]

where \( N_s(r) \) is the cumulative number of shocks up to radius \( r \), and \( n_s \) measures the local differential number of new, emerging shocks. This formalism emphasizes the importance of the number of shocks and their spatial distribution; compared to the traditional emission measure approach, it should provide more physically motivated constraints for wind-shock X-ray production in massive stars.

In particular, the analysis below of multiple, instability-generated shocks assumes a power-law scaling for \( n_s \) (see equation 25); but for the above single-shock model, this just takes the form of a Dirac delta-function, \( n_s = \delta(r - r_s) \). The associated X-ray luminosity is then given by integration of the emissivity \( \eta_s \) over solid angle and volume,

\[
L_s = 16\pi^2 \int C_s f_{s \text{ss}} r \delta(r - r_s) \rho^2 r^2 dr. \tag{18}
\]

Upon trivial evaluation over the delta function, comparison with the scalings (14c) and (15b) yields the identifications,

\[
C_s = \frac{3}{\pi} \frac{\Lambda_m(T_s) f_{s \text{ss}}}{\ell_s}, \tag{19}
\]

and

\[
f_{s \text{ss}} = \frac{1}{1 + r_s/\ell_s}. \tag{20}
\]

This thus now sets a bridging law at the level of an individual shock, with \( f_{s \text{ss}} \) characterizing the fraction of single-shock emission measure that actually contributes to radiative emission. The adiabatic limit \( \ell_s \gg r_s \) gives \( f_{s \text{ss}} \approx 1 \) and so a density-squared scaling for the emissivity (16); the radiative limit reduces this by the small factor...
radius, $\eta$ the total X-ray emissivity to be an  
the general scalings for the X-ray luminosity, one can again take  
section 4 and Owocki & Cohen (2001, hereafter OC01). But to derive  
directional dependence of the emissivity and optical depth; see Sec-

be reasonably well described by a globally spherical wind emission  
associated with a complex, 3D, stochastic wind structure (Dessart  
and flow speed $v_c$ as following some specified radial function, e.g. a power law.

3 X-RAYS FROM INSTABILITY-GENERATED  
EMBEDDED WIND SHOCKS

3.1 Exospheric scaling for $L_x$  
To model the X-ray emission from multiple, embedded wind shocks, let us next write the X-ray luminosity in a fully general, 3D form  
that accounts for possible directional dependencies in emission and absorption,

$$L_x = \int d^3r \int d\Omega \eta_x(r, n) e^{-\tau(r, n)} ,$$  
(21)

where the optical depth $\tau(r, n)$ accounts for bound-free absorption  
of X-rays emitted at location $r$ in the observer direction $n$.

In principle, instability-generated, X-ray emitting shocks will be  
associated with a complex, 3D, stochastic wind structure (Dessart  
& Owocki 2003). But upon averaging over small scales, this can be  
reasonably well described by a globally spherical wind emission  
model (OC99). In modelling line emission, accounting for the  
observed Doppler shift from wind expansion still requires including a  
directional dependence of the emissivity and optical depth; see Section  
4 and Owocki & Cohen (2001, hereafter OC01). But to derive the  
general scalings for the X-ray luminosity, one can again take  
the total X-ray emissivity to be an isotropic function of the local radius, $\eta_x(r)$. Moreover, given the weak to moderate importance of  
absorption in all but the densest winds, its overall role in scaling  
relations can be roughly taken into account through a simple exo-
spheric approximation (OC99; Leutennegger et al. 2010), for which the  
integrations over solid angle and volume in (21) reduce to just  
a single integration in radius,

$$L_x \approx 16\pi^2 \int_{R_o}^{\infty} \eta_x(r) r^2 dr ,$$  
(22)

where the integral lower bound is taken from the larger of the X-ray  
onset radius and the radius for unit radial optical depth, i.e. $R_o \equiv \max [R_{o1}, R_{o2}]$. For a wind with mass-loss rate $\dot{M}$ and flow speed $V_\infty$, this radius for transition from optically thick to thin X-ray emission is given by

$$R_{o1} \equiv \tau_a R_s \equiv \frac{\kappa_{xb} \dot{M}}{4\pi V_\infty} \approx 25 R_\odot \frac{M_{-6}}{E_{kev} V_{1000}} ,$$  
(23)

where $\tau_a$ is a characteristic wind optical depth to the stellar surface  
radius $R_s$, and the latter equalities use the scaling (9) for the bound-

$^{6}$For instability-generated wind structure, X-rays can arise from collisions  
between clumps that have been compressed to some fraction of the wind  
volume (Feldmeier et al. 1997b), implying then a higher input density  
that would lower the cooling length. For simplicity, the analysis here does not  
account for this possibility, but it could enhance the importance of radiative  
cooling, leading to an even larger effective ratio $\kappa_{cs}/\kappa_{bf}$ of cooling to absorption.

Figure 2. Ratio $R_s/R_o$ of adiabatic radius to shock onset radius, assuming  
$R_o = 1.5 R_s$, and a post-shock temperature $T_{kev} = 0.5$. To construct the plot,  
stellar parameters were taken from tables 1–3 of Martins et al. (2005) ($T_{eff}$,  
$L_{bol}$, $R_*, M_*$) for luminosity classes V, III and I (marked, respectively, by  
circles, squares and triangles). The terminal speeds are computed as $V_\infty = 2.6V_{tec}$, and mass-loss rates are from Vink, de Koter & Lamers (2000).

3.2 Bridging law with thin-shell mixing  
Previous analyses using the density-squared emissivity (16) (e.g.  
OC99; OC01) have directly parametrized the X-ray filling factor $f_\eta$ as following some specified radial function, e.g. a power law.
But instead let us now parametrize the shock-number distribution in (17) by an analogous power law,

$$n_s(r) \equiv n_{s0} \left( \frac{R_s}{r} \right)^{\beta}; \quad r > R_s, \quad (25)$$

with $n_{s0} = 0$ for $r < R_{so}$. Both instability simulations (Owocki & Puls 1999; Runacres & Owocki 2002; Dessart & Owocki 2003) and X-ray profile fitting and He-like / H ratios (Cohen et al. 2006, 2010; Leutenegger et al. 2006) suggest an initial onset for shock formation around $R_{so} \approx 1.5 R_s$.

Applying (25) in (17), the analysis in Section 2.4 provides a more physical model of shock X-ray emission that accounts explicitly for the effects of radiative cooling. However, it still does not account for any thin-shell mixing. The inherent thinness ($\ell \ll r$) of radiative shock cooling zones makes them subject to various thin-shell instabilities (Vishniac 1994), which in numerical simulations lead to highly complex, turbulent shock structure (e.g. Walder & Folini 1998). Parkin & Pittard (2010) discuss how the inherently limited spatial resolution of radiative shock simulations leads to a ‘numerical conduction’ that transports heat from high- to low-temperature gas, resulting in a severe, but difficult-to-quantify reduction in the X-ray emission.

While the specific mechanisms within hydrodynamical simulations may indeed depend on such numerical artefacts, the perspective advocated here is that such an overall reduction in X-ray emission is likely a natural consequence of the turbulent cascade induced by the thin-shell instability; this should lead to substantial physical mixing between cool and hot material, with the softer and more efficient radiation of the cooler gas effectively reducing the emission in the X-ray bandpass.

Pending further simulation studies to quantify such mixing and X-ray reduction, we make here the plausible ansatz that the reduction should, for shocks in the radiative limit $\ell/r \ll 1$, scale as some power $m$ of the cooling length ratio, $(\ell/r)^m$. To ensure that the mixing becomes ineffective in the adiabatic limit – for which the cooling term in (28) in key asymptotic limits – we recast the bridging law (20) for the shock volume filling factor $f_x$, in the generalized form,

$$f_x \approx \frac{1}{(1 + r/\ell)_{1+m}^2} = \frac{1}{(1 + \kappa_s \rho r)_{1+m}^2}, \quad (26)$$

with the level of mixing now controlled by a positive value of the mixing exponent $m$. To simplify notation in the analysis to follow, we have dropped here the subscripts (‘s’ or ‘w’) for the quantities (e.g. $\ell$, $r$, $\kappa_s$, $\rho$) on the right-hand-side. Recalling that the cooling coefficient $\kappa_s$ takes the scalings given in equations (8a)–(8c), we can use the adiabatic radius $R_{a}$ from (24) to characterize the asymptotic regimes for (26).

If $R_{a} < R_{o}$, then the shocks are adiabatic throughout the wind, and we recover the standard density-squared emissivity (16) with a specified volume filling factor set by $f_x = n_{s0}$. If $R_{a} > R_{o}$, this adiabatic scaling still applies in the outer wind, $r > R_{a}$; but in the inner regions $R_{o} < r < R_{a}$, the cooling term in the denominator of (26) dominates, giving the emissivity (16) now a reduced dependence on density,

$$\eta_x \approx C_p n_o \rho_{1-m}^2/(\kappa_s r)_{1+m}^2; \quad R_{o} < r < R_{a}, \quad (27)$$

Without mixing ($m = 0$), the density dependence is thus linear, but with mixing ($m > 0$), it becomes sub-linear.

As noted above, absorption is a modest effect in even dense O-star winds, with $r_*$ at most of the order of unity, implying then that $R_{a} \ll R_{o}$ (Cohen et al. 2010). But the stronger coefficient $(\kappa_s/\kappa_{so} = R_{a}/R_{o} \approx 25)$ for radiative cooling means that such winds typically have $R_{a} > R_{o}$, implying that most shocks remain radiative well above the wind acceleration region where they are generated.

### 3.3 $L_x$ scalings with $M/V_\infty$

Let us now turn to the scalings for the overall X-ray luminosity. Applying the emissivity (16) and filling factor (26) to the exospheric model (22) with the power-law shock distribution (25), we find

$$L_x \approx 16\pi^2 \rho C_x \int_{R_{a}}^{\infty} \frac{n_{s0} \rho^2}{(1 + \kappa_s \rho r)^{1+m}} r^2 dr$$

$$= C_p \left[ \frac{M}{V_\infty} \right]^2 \int_{R_{a}}^{\infty} \frac{dr}{r^{(p)(1-n)_{1+m}}(r + R_{a})^{1+m}}, \quad (28)$$

where $C_p \equiv C_p n_{s0} R_{o}^2$ and the latter equality uses mass conservation to cast the integral in terms of the scaled wind velocity, $w(r) \equiv r/(r + R_{a})$. This is generally taken to have a ‘beta-law’ form $w(r) = (1 - R_{a}/r)^{\beta}$, with the canonical case $\beta = 1$ giving $r w = r - R_{a}$, and the constant-speed case $\beta = 0$ giving $r w = r$. For these special cases, Section 5 gives some results for full integrations of (28).

But even for a generic velocity law $w(r)$, we can readily glean the essential scalings of the $L_x$ from (28) in key asymptotic limits of adiabatic versus radiative shocks.

#### 3.3.1 Adiabatic shocks in optically thin wind

First, for low-density winds with optically thin emission from adiabatic shocks, $R_{a} < R_{o} < R_{a}$, we can effectively drop the $R_{a}$ term in the denominator, and set the lower bound of the integral to the fixed onset radius, $R_{o} = R_{o}$, yielding

$$L_x \approx C_p \left[ \frac{M}{V_\infty} \right]^2 \int_{R_{o}}^{\infty} \frac{dr}{r^{(p)(1-n)_{1+m}}}; \quad R_{o} < R_{a} < R_{o}. \quad (29)$$

Since the resulting integral then is just a fixed constant that is independent of $M$, the X-ray luminosity recovers the standard adiabatic scaling $L_x \sim (M/V_\infty)^2$.

#### 3.3.2 Radiative shocks in optically thin or thick wind

For high-density winds with radiative shocks and so $R_{a} < R_{o}$, the radiative effect now dominates its term, and so can be pulled outside the integral. Rescaling the remaining integrand in terms of the initial radius $R_{o}$, we find

$$L_x = C_p \left[ \frac{M}{V_\infty} \right]^2 \frac{R_{o}^{n-1}}{R_{o}^{n+2}} \int_{1}^{\infty} \frac{dr}{r^{(p)(1-n)_{1+m}}}; \quad R_{a} < R_{a}, \quad (30)$$

where again the integral is now essentially independent of $M/V_\infty$.

For the intermediate-density case in which the radiative shock emission is optically thin, $R_{a} < R_{o} < R_{a}$, the integration lower limit is fixed at the onset radius, $R_{o} = R_{o}$, which is independent of $M/V_\infty$. But since $R_{o} \sim M/V_\infty$, the overall scaling is $L_x \sim (M/V_\infty)^{1-m}$.

For the case of very dense, optically thick winds with radiative shocks, $R_{a} < R_{a} < R_{a}$, the lower boundary at $R_{a} = R_{a}$ gives the residual integral an additional dependence on $R_{a}^{n-2-p}$; since $R_{a}$ too scales with mass-loss rate, the dependence on the mixing index $m$ cancels. At the radius $R_{a}$, the cooling length ratio $\ell/r$ is always the same, implying that in optically thick winds the observed radiative shock emission is likewise fixed for any mass-loss rate. The luminosity scaling thus becomes independent of mixing, scaling just with shock-number index as $L_x \sim (M/V_\infty)^{-p}$.
This is the same scaling found in OC99 for optically thick winds, but with *adiabatic* shocks. Indeed, OC99 argued that assuming a filling factor \( f_\ast \sim r^{-0.4} \) could give a sub-linear dependence on mass-loss rate, \( \dot{M} \sim M^{0.6} \), and so possibly reproduce the \( L_\ast \sim L_{\rm bol} \) relation. Subsequent analysis of X-ray line profiles observed from *Chandra* and *XMM–Newton* have shown, however, that optical depth effects are quite moderate even for O-stars like \( \zeta \) Puppis with quite dense winds (Cohen et al. 2010). The bulk of O-star winds are simply too low density for this optical thickness scaling to apply, and so this cannot be the explanation for the \( L_\ast \sim L_{\rm bol} \) relation.

### 3.3.3 Summary of asymptotic scalings

To summarize, power-law shock-number models with thin-shell mixing have the asymptotic scalings,

\[
L_\ast \sim \left[ \frac{M}{V_\infty} \right]^2 \quad ; \quad R_1 < R_\ast < R_0 \quad ; \quad \text{adiabatic, thin} \quad (31a)
\]

\[
\sim \left[ \frac{M}{V_\infty} \right]^{1-m} \quad ; \quad R_1 < R_0 < R_\ast \quad ; \quad \text{radiative, thin} \quad (31b)
\]

\[
\sim \left[ \frac{M}{V_\infty} \right]^{1-p} \quad ; \quad R_0 < R_1 < R_\ast \quad ; \quad \text{radiative, thick} \quad (31c)
\]

where the progression represents a trend of increasing \( M/V_\infty \). The first applies for weak winds, for example from early B main-sequence stars, which typically show weak X-ray emission, with \( L_\ast \) well below \( 10^{-3} L_{\rm bol} \) (Cohen et al. 1997). The middle scaling for intermediate-density winds is the most relevant for the bulk of O-type stars found to follow the \( L_\ast \sim L_{\rm bol} \) relation. The last applies only to the strongest winds, e.g. very early O supergiants like HD93129A, for which analysis of X-ray line profiles show moderate absorption effects,\(^7\) with \( \tau_\ast \gtrsim 1 \) (Cohen et al. 2011).

### 3.4 Link between \( \dot{M} \) and \( L_{\rm bol} \) scaling

As noted in the introduction, and summarized in Fig. 1, straightforward application of CAK wind theory implies a direct dependence of mass-loss rate on luminosity that scales as \( \dot{M} \sim L_{\rm bol}^{1/3} \), where \( \alpha \approx 0.6 \) is the CAK power index; for the bulk of O-type stars with intermediate-density winds and thus \( L_\ast \sim M^{1-m} \), reproducing the observed \( L_\ast \sim L_{\rm bol} \) relation thus simply requires a mixing exponent \( m = 1 - \alpha \approx 0.4 \).

More generally, let us now consider how this requirement is affected if one accounts also for a secondary dependence on stellar mass \( M \), which in turn can give a further indirect dependence on \( L_{\rm bol} \). Specifically, within CAK wind theory, \( \dot{M} \sim M^{1-m/\alpha} \) and \( V_\infty \sim M^{1/2} \), and so if we in turn assume from stellar structure a mass–luminosity dependence \( M \sim L_{\rm bol}^{s/3} \), where \( s \approx 1/3 \) (e.g. Maeder 2009, p. 360), we find

\[
\log \left( \frac{M}{V_\infty} \right) \sim \frac{2 - s(2 - \alpha)}{2\alpha} \log(L_{\rm bol}) \sim \log(L_\ast) \frac{1}{1-m}, \quad (32)
\]

where the latter relation makes use of the scaling (31b).

---

\(^7\) Indeed, this star could be viewed as a transitional object to the WNH-type Wolf–Rayet stars, for which absorption effects should strongly attenuate any X-rays from instabilities in the wind acceleration region; see Section 6.

\(^8\) Of course, the terminal speed also depends on stellar radius as \( V_\infty \sim 1/\sqrt{R_*} \), but the diverse luminosity classes of X-ray emitting O-stars makes it difficult to identify any systematic dependence on \( L_{\rm bol} \) that might influence the overall \( L_\ast \sim L_{\rm bol} \) relation.

---

Reproducing the empirical \( L_\ast \sim L_{\rm bol} \) relation thus now requires

\[
m = \frac{2(1 - \alpha) - s(2 - \alpha)}{2 - s(2 - \alpha)}. \quad (33)
\]

Accounting for a stellar structure scaling \( s \approx 1/3 \) with a CAK index \( \alpha \approx 0.6 \) thus now requires a mixing exponent \( m \approx 0.22 \), somewhat smaller than the \( m \approx 0.4 \) required if one assumes no systematic mass–luminosity scaling (i.e. \( s = 0 \)).

### 4 EFFECT OF SHOCK COOLING ON X-RAY LINE PROFILES

#### 4.1 Basic formalism

Observations by *XMM–Newton* and *Chandra* of spectrally resolved, wind-broadened X-ray emission lines from luminous OB stars provide a key diagnostic of the spatial distribution of X-ray emission and absorption in their expanding stellar winds (Ignace & Gayley 2002; Oskinova, Feldmeier & Hamann 2006; Owocki & Cohen 2006). In particular, the relatively weak blue–red asymmetry of the observed emission lines indicates modest wind optical depths, \( \tau_\ast \sim 1 \), while the overall width constrains the spatial location of the emission within the expanding velocity law. For the usual density-squared emission model with a prescribed (power law) spatial variation in volume filling factor \( f_\ast \), fits to observed X-ray lines are typically consistent with a standard \( \beta \approx 1 \) velocity law and an \( f_\ast \) that is spatially nearly constant, corresponding to \( p \approx 0 \) within the adiabatic scaling implicit in the emission measure analysis (Kramer, Cohen & Owocki 2003; Cohen et al. 2006). In the discussion below, we refer to these profiles – plotted in black in Fig. 3 – as ‘observationally favoured’.

Within the perspective discussed here that shocks within most O-star winds are likely to be radiative instead of adiabatic, let us...
now examine how inclusion of radiative cooling affects X-ray line profiles. Following OC01, the directional Doppler shift of the X-ray line emission within the expanding wind is modelled through a line emissivity \( \eta_l(r, \mu) \) at an observer’s wavelength \( \lambda \) along direction cosine \( \mu \) from a radius \( r \). The resulting X-ray luminosity spectrum \( L_x \) is computed from integrals of the emission over direction and radius, attenuated by bound-free absorption within the wind (cf. OC01, equation 1),

\[
L_x = 8\pi^2 \int_{-1}^{1} d\mu \int_{R_i}^{\infty} dr r^2 \eta_l(\mu, r) e^{-\tau(\mu, r)}. \tag{34}
\]

The absorption optical depth \( \tau(\mu, r) \) is evaluated by converting to ray coordinates and then integrating for each ray with a fixed impact parameter from the local position to the observer. For the standard \( \beta = 1 \) velocity law, the integrals are analytic, with overall scaling in proportion to \( \tau_s \equiv R_s/R_\ast \).

In principle, this integrated optical depth can be affected by the ‘porosity’ associated with optically thick clumps or anisotropic ‘pancakes’ (Feldmeier, Oskinova & Hamann 2003), with potential consequences for interpreting the asymmetry of X-ray line profiles in terms of the wind mass-loss rate (Oskinova et al. 2006). But for the optically thin (\( \tau_s \ll 1 \)) or marginally optically thick (\( \tau_s \sim 1 \)) lines considered here, individual clumps should be optically thin (Owocki & Cohen 2006; Sundqvist et al. 2012), and so such effects are not important for the discussion below, which focuses on the overall profile width.

4.2 Scaling analysis

As noted, applications of this OC01 formalism assuming a density-squared emission show that a spatially constant X-ray volume filling factor \( f_v \) gives generally quite good fits to observed X-ray lines. To examine the effect of radiative cooling, let us now apply the more general bridging-law scalings of (16) and (26), assuming the simple power-law form for the shock number (25). The spatial integration thus takes the same form as the exospheric result (28), except that absorption is now treated explicitly by the exponential optical depth term in the integrand, with the radial lower bound fixed at the X-ray onset radius, \( R_i = R_\ast \).

We can again infer basic scaling results by inspection of this integral in the limits of adiabatic versus radiative shocks. For the adiabatic case, this follows the scaling in (29), with radial dependence \( 1/w^2 r^{1+m} \). To match observed profiles, such adiabatic emission models require constant \( f_v = n_a \), with the zero power-law exponent \( p = 0 \), implying a \( 1/w^2 \) dependence of the integrand.

By contrast, the radiative limit follows the scaling in (30), with direct radius dependence \( 1/w r^{1+m} \), and velocity dependence \( 1/w^{1-m} \) that is weaker than the \( 1/w^2 \) of the adiabatic model. Focusing first just on the former, we see that reproducing the \( 1/w^2 \) integrand needed to fit observed profiles would now require \( p = 1 + m \). In practice, to compensate for the weaker inverse-speed dependence, fitting the observed profile width requires a somewhat steeper shock-number decline, as we now quantify.

4.3 X-ray line profiles from radiative shocks

For a \( \beta = 1 \) velocity law with \( R_\ast = 1.5 R_e \) and various specified values of the exponents \( p \) and \( m \), Fig. 3 plots normalized X-ray line profiles for optically thin (\( \tau_s \ll 1 \); left) and marginally optically thick (\( \tau_s = 1 \); right) lines. In both cases, the black curve represents the adiabatic, constant-filling-factor (\( p = 0 \)) model that gives generally good fits to observed profiles.

The other curves show results for radiative shocks. Without mixing, the red curve for radiative shocks with constant shock number (\( p = 0 \)) is far too broad; fitting the favoured black profile now requires a \( p = 1.5 \) (blue curve), which is even steeper than the predicted \( p = m + 1 \) = 1 needed to compensate for the weaker direct radial scaling. With mixing exponent of \( m = 0.4 \), we find that a \( p = 1.5 \) shock-number model gives profiles (not shown) that are still somewhat too broad. But with a somewhat steeper number exponent \( p = 2 \), the blue curve again nearly reproduces the black curve.9

An overall conclusion is thus that, for radiative shock models with mixing at a level needed to reproduce the \( L_x \sim \dot{M}_{\ast} \) relation, matching observed X-ray emission requires a steep radial decline (\( p \approx 2 \)) in shock number above the onset radius.

4.4 Decline of \( L_x \) in optically thick winds

Such a steep radial drop off in shock number has important implications for the scaling of X-ray luminosity for the highest density stars that become optically thick to bound-free absorption, i.e. with \( \tau_s > 1 \) and thus \( R_i > R_s \). Namely, from the scaling given by (31c), we see that taking \( p = 2 \) implies that the X-ray luminosity for such optically thick winds should now decline inversely with mass-loss rate, \( L_x \sim M^{p-2} \sim 1/M \). If the X-ray emission is concentrated near an onset radius within the wind acceleration zone, the bound-free absorption by the overlying, optically thick wind significantly attenuates the net X-rays seen by an external observer. For early-O-type supergiants with dense winds, this can lead to a reduced X-ray luminosity, but because the overall decline of bound-free opacity with X-ray energy, the observed spectrum can be hardened. The recent analysis of X-rays from the O2If star HD 93129A provides a potential example approaching this limit (Cohen et al. 2011).

5 SCALING RESULTS FOR FULL EVALUATION OF \( L_x \) INTEGRAL

5.1 Constant-speed wind with \( \beta = 0 \)

As a supplement to the asymptotic \( L_x \) scalings given in Section 3.3, let us finally consider full solutions for the general integral (28). For a constant speed model with \( \beta = 0 \) and thus \( w(r) = 1 \), general analytic integration is possible in terms of the incomplete beta function. But the typical properties can be more simply gleaned by examining the special case \( p = 1 \), for which the integral in (28) takes the simpler analytic form

\[
L_x = \frac{C_p \kappa_{\infty}^2}{16\pi^3 m(1-m)} \left[ 1 + m R_s/R_i \left( 1 + R_s/R_i \right)^m - 1 \right]. \tag{35}
\]

wherein the square-bracket factor sets the scalings with \( \dot{M}/V_\infty \), with the preceding terms just fixing the overall normalization. For low-density, optically thin winds, the initial radius is fixed to the onset radius, so that \( R_s/R_i \sim R_s/R_\ast \sim \dot{M}/V_\infty \). For \( R_s/R_i \ll 1 \), expansion of the square-bracket term recovers the adiabatic scaling \( L_x \sim (\dot{M}/V_\infty)^2 \) of (31a), while for \( R_i/R_s \gg 1 \), it becomes proportional to \( R_s^{1+m} \) and so recovers the radiative, optically thin scaling (31b). For high-density, optically thick winds, the ratio

9 Also not shown here are profiles computed for \( p = 2 \) and the alternative mixing exponent value \( m = 0.22 \), which we find also give close agreement with the black curves.
assume, following (23) and (24), \( f_{i,a} = R_i/R_o = \kappa_{bi}/\kappa_o \approx 1/100. \) For very large \( R_o > R_i/f_{i,a} \approx 100R_o, \) we thus have \( R_i > R_o, \) leading to a declining \( L_x, \) as predicted by the optically thick wind scaling \( L_x \sim R_i^{1-\beta} \) from (31c).

But for moderately dense winds, with \( R_o < R_i < R_i/f_{i,a} \) (between the vertical lines in the figure), the increasing \( L_o \) approaches the power-law variation \( R_i^{1-m} \) predicted by (31b). In particular, the black curve with \( p = 2 \) and \( m = 0.4 \) represents the preferred model with sub-linear scaling in \( L_x, \) and thus in \( M/V_\infty, \) implying a nearly linear scaling of \( L_x \) with \( L_{bol}. \)

Specifically, for typical values for stellar radius \( (R_o \approx 10-20R_\odot) \) and wind terminal speed \( (V_\infty \approx 2000 \text{ km s}^{-1}), \) this intermediate-density regime with \( L_x \sim L_{bol} \) applies to wind mass-loss rates that range from below \( 10^{-7} \text{ M}_\odot \text{yr}^{-1} \) to a few times \( 10^{-6} \text{ M}_\odot \text{yr}^{-1}; \) this essentially encompasses the entire O-star spectral range for which the \( L_x \sim 10^{-3}L_{bol} \) relation is found to hold.

### 6 CONCLUDING SUMMARY

The central result of this paper is that, in the common case of moderately dense O-star winds with radiative shocks \( (R_o > R_i), \) thin-shell mixing can lead to this sub-linear scaling of the X-ray luminosity with the mass-loss rate, \( L_x \sim (M/V_\infty)^{1-m}. \) Depending on the secondary scalings of wind density with bolometric luminosity, one finds that \( m \approx 0.2-0.4 \) can give roughly the linear \( L_x - L_{bol} \) law that is empirically observed for O-star X-rays. Further simulation work will be needed to see if such mixing exponent values are

---

**Figure 4.** Normalized X-ray luminosity \( L_x \) versus adiabatic radius \( R_o \) scaled by shock onset radius \( R_o = 1.5R_o, \) for winds with a standard \( \beta = 1 \) velocity law, and selected, labelled values for the parameters \( p \) and \( m. \) The dashed, dot-dashed and dotted lines show the expected scalings for, respectively, adiabatic, radiative and thin-shell-mixed shocks. As discussed in the text, the variation in \( R_i \) represents a proxy for the wind density parameter \( M/V_\infty, \) ranging from low-density, adiabatic shocks on the far left, to high-density, optically thick winds with radiative shocks on the far right: the optically thick turnover at \( R_o/R_o > 100 \) applies for observed X-ray energies that are about a factor of 2 higher than the shock energy. The intermediate-density case with optically thin, radiative shocks follows the \( L_x \sim M^{1-m} \) scaling that reproduces the empirical \( L_x \sim L_{bol} \) relation if \( m \approx 0.4. \) The upper axis uses the analysis from Fig. 2 to mark the corresponding spectral class for main sequence stars; for higher luminosity giants and supergiants, the associated spectral class sequence would shift to the right.
appropriate, and indeed to test the validity of the basic mixing exponent ansatz.

But in the course of exploring this idea of thin shell mixing, the analysis here has lead to several interesting secondary results with validity and implications that are largely independent of mixing or any specific model for it. A summary list includes the following.

(i) In contrast to previous analyses that invoked a density-squared emission measure description for shock production of X-rays, we derive here a more general bridging law showing how the density-squared scaling of adiabatic shocks transitions to a single density scaling for radiative shocks.

(ii) For radiative shocks, the X-ray volume filling factor is not fixed (as is commonly assumed), but is reduced by the narrow extent of the shock layer, \( f_x \sim \ell / r \).

(iii) For nearly all O-stars, the large radiative–adiabatic transition radius, \( R_\text{c} \gg R_* \approx 1.5 R_\odot \), implies that instability-generated shocks in the wind acceleration region should follow the radiative scaling, giving the X-ray luminosity a linear scaling with mass-loss rate, \( L_x \sim M/V_\infty \).

(iv) For low-density winds of lower luminosity (early B) stars, shocks should indeed become adiabatic, implying then a steep \( (L_x \propto M^2) \) decline of X-ray luminosity, as is in fact generally found for single, non-magnetic, early B-type stars, for which the inferred X-ray emission measure often approaches that of the full wind (Cohen et al. 1997, 2008)

(v) Matching observed X-ray emission lines with such models of radiative shocks with or without thin-shell mixing requires the shock number to have a moderately steep decline above the X-ray onset radius, with power-law exponent \( p \approx 1.5–2 \).

(vi) This in turn implies that the scaling of X-ray luminosity for dense, optically thick winds should saturate and even decline with increasing mass-loss rate.

This last result on X-ray absorption is perhaps not too relevant for most of the O-stars following the \( L_x \sim L_\text{bol} \) relation, for which optical depth effects are weak to marginal. But it can become important for the dense, moderately optically thick winds of extreme, early O-stars like HD 93129A, which can be viewed as a transitional object to the WNH-type Wolf–Rayet stars (Smith & Conti 2008). More generally, the high density of Wolf–Rayet winds imply that absorption should strongly attenuate X-rays from any instability-generated shocks in their wind acceleration region. As such, the observed hard X-rays seen from Wolf–Rayet stars like WR6 (EZ CMa) seem unlikely to be explained by this standard model of LD1 shocks (Oskinova et al. 2012). This also has potential implications for interpreting observed X-rays from very massive stars that have \( L_x \sim 10^{-5} L_\text{bol} \) despite having very high wind optical depths (Crowther et al. 2010), and whether these might instead originate from wind–wind collisions of close, undetected binary companions.

Indeed, the mixing ansatz in this paper could also be applied to model X-ray emission from CBW, and their \( L_x \) scaling with orbital separation. Wide binaries with adiabatic shocks should still follow the usual inverse distance scaling, as directly confirmed by observations of multiyear-period elliptical systems like WR140 and \( \eta \) Carinae (Corcoran 2012). But in close, short (day to week) period binaries with radiative shocks (Antokhin et al. 2004), mixing could reduce and limit the effective X-ray emission from the wind collision (Parkin & Pittard 2010), and thus help clarify why such systems often hardly exceed the \( L_x \approx 10^{-7} L_\text{bol} \) scaling found for single stars (Oskinova 2005; Corcoran 2012; Gagné et al. 2012).

Finally, in addition to exploring such effects in CBW, a top priority for future work should be to carry out detailed simulations of the general effect of thin-shell mixing on X-ray emission, and specifically to examine the validity of this mixing-exponent ansatz for modelling the resulting scalings for X-ray luminosity.

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REFERENCES

Thin-shell mixing and O-star X-rays


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