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Advising a Precollege Curriculum Project

Stephen B. Maurer and William McCallum

We two are members of the content advisory group for the Core-Plus Mathematics Project (CPMP), one of several projects funded by the National Science Foundation (NSF) to develop a whole secondary school mathematics curriculum. Serving in this advisory group has been a fascinating experience. We hope we have helped the project; we know we have learned a lot in the process. We write this article to explain what is involved, to suggest to readers that this is a very worthwhile sort of activity for professional mathematicians, and to encourage readers to seek out similar opportunities.

What Is Core-Plus Mathematics?

Core-Plus consists of a three-year core program intended for a wide range of high school students, plus a fourth-year course continuing the preparation of students for college mathematics. It is published by Glencoe/McGraw-Hill under the title *Contemporary Mathematics in Context: A Unified Approach*. Key features of the curriculum include: teaching algebra and geometry every year along with important new topics from statistics and discrete mathematics, emphasizing mathematical modeling and applications, and teaching students to solve more challenging problems.

We are part of the content advisory group for the second edition. Other members are Doris Schattschneider of Moravian College, an expert on geometry, Richard Scheaffer of the University of Florida, a Fellow of the American Statistical Association, and Deborah Hughes Hallett of Harvard University and the University of Arizona, founder of the Harvard Calculus Consortium.

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What We Do

Core-Plus invited us to contribute as content specialists and critics of the actual writing. We have also tried to give advice on the overall goals and coverage of the program. We discuss some of the things we have been critical about later on. The group meets with the CPMP development team once a year, in late May or early June. In advance of the meeting we read the current drafts of units that were worked on during the past year and comment about them, both in the large and in the small. After the meeting, there may be further correspondence between individual consultants and individual writers, for units under revision about which there were special concerns.

How We Got Involved

Both of us have been involved in curricular development at the college level. The first of us has been especially interested in discrete mathematics, so he had his ears perked up for precollege developments. He got further involved when his children's school district adopted the Interactive Mathematics Project (IMP) curriculum (another one of the NSF-funded projects); initially he had very mixed feelings about IMP but eventually became mostly positive. He first saw copies of the Core-Plus books on a visit to DIMACS and was immediately intrigued by how much discrete mathematics was in them. He had met the discrete mathematics writer for Core-Plus at various meetings and expressed interest in being part of the project. Eventually this resulted in an invitation to the content advisory group.

The second of us has worked in calculus reform since the late 1980s, and has recently been working on a college algebra text. This has led to an interest in how algebra is taught in high school. He met the Core-Plus authors at a National Academies meeting.

Why It's Fun

It's always fun to be with a lively thoughtful group of new colleagues, both fellow consultants and the developers of the materials, who are doing valuable work and who bring experiences and insights to the table that are different from yours. Part of the fun has been the interesting debates we have had (see below). Part of it has been the experiences and good humor of the group. In particular, it has been heartening to meet the teachers who have been working with these materials and to hear their stories of how these materials work in the classroom, which lies somewhere between the rosy vision of the developers and the dark vision of their detractors. The dedication and common sense of these teachers is remarkable.

What We've Taught

To explain what we've taught, it is necessary to say a bit about the position of this project in the math wars. (We do this with some reluctance, because it is not the purpose of this report to take sides in that war; our purpose is to report on what we have done and to suggest that the role we have played would be of interest to mathematicians of any persuasion.) The NSF-funded secondary curriculum projects referred to earlier have received a lot of criticism from some mathematicians and from organized Web-based groups. The views of the advisory group members vary, but it would probably be accurate to say that each of us agrees with some of the criticisms and disagrees with others.

One thing that all sides in the math wars agree on is: more students should *understand* what they are doing. The disagreement comes in how this is to be achieved. Core-Plus initially took an approach to algebra that, in Courses 1 and 2, emphasized tables, graphs, real-world examples, and verbal descriptions, at the expense of some traditional topics and skills. Then the algebra ratcheted up substantially in Course 3. One of the things we have helped the writers do is even out the introduction of algebraic methods.

Other things we have done are: identify mathematical statements that are sufficiently vague as to be misleading or wrong, object to unnecessary neologisms, note terminology whose use is contrary to its use in more advanced mathematics, and ask for more attention to definitions—both including them and helping students to understand their important role. We have suggested adding or deleting various subjects and problems.

What We've Learned

There is more to math education than knowing the math. Yes, everybody knows that, or says they do, but working with a curriculum project gives this statement a richer meaning than one can otherwise imagine.

First, there are all sorts of constraints on the ground. At each yearly meeting, several school teachers are in attendance, and they report on the past year and comment on suggestions we make. Who would have thought that an important issue in deciding the order of units is the density of class interruptions at the corresponding time of year—vacations, assemblies, special state tests, weather, etc.? Then there are problems with absenteeism, and attitudes of other teachers, administrators, parents. How districts buy textbooks also has an effect. In some districts you can bank this year's money for buying texts; in some you can't. In some places this means that they won't be able to consider switching to the second edition until several years after all four new books are out.

Second, introducing a curriculum is an immense project. Each of us has found writing the text for a single course a huge project, but Core-Plus is four years of integrated courses. Plus teacher editions, assessment resources, and software. Plus systematic evaluation at all stages. Plus teacher workshops. Plus dealing with publishers. And the public. And so on. We tip our hats to people who can pull this off. It's easy for us to criticize here and there, but these authors have to put all the pieces together.

Finally, what we as mathematicians might think is most interesting, or most challenging, doesn't always jibe with what teachers and students think. We thought that the early discrete mathematics units might be disliked, by both teachers and students, because the material is unfamiliar and doesn't meet traditional expectations about what is mathematics. In fact, students tend to like it as relatively easy and a nice break from other stuff. But then, if it isn't on the state test (and it usually isn't), and the class is behind schedule, the discrete mathematics material is the first to go.

Other things we have learned are more mathematical. Core-Plus has a wealth of fascinating examples. We are not statisticians, so we have learned many things from the statistics examples—fascinating data and even new tests. For instance, suppose you want to test whether two samples are from the same distribution, but you know nothing yet about normal curves, t-tests, and so forth. There are ways to get at this issue by taking many random redivisions of the data.

The first-named author is an expert in discrete mathematics, and yet he has found a fair amount of the information in the CPMP discrete mathematics units to be new to him (examples, applications and vignettes). The Core-Plus authors have looked far and wide for useful information.

We think the most fascinating things we have learned are about pedagogy. For instance, what is the role of a textbook?

At a university, students buy their books, and advanced students may even keep their books and refer to them in later courses and later in life. Thus textbooks also serve as a reference. University students are also expected to have the intellectual maturity to understand complex logical exposition. But in high school, students don't generally keep their books, they are just beginning to learn the conventions of mathematicians, and they learn in roundabout ways. Consequently, the Core-Plus books are not meant as references—far from it. Once we understood this, we understood many of the design decisions made by the authors. In general, there is a tension between precision and clarity—what is clear at a certain stage of conceptual development may be far from precise, and curriculum developers have the daunting task of trying to find clarity for a wide range of students.

That doesn't mean we accepted everything about the textbook design decisions. For instance, maybe students don't need a complete index, especially one that covers all four volumes, but teachers and the mathematical public sure do. So we asked for more and better indexes. Also, some attempts to balance the tension between precision and clarity backfire and make things worse. Many times we have urged the writers to try again.

A Specific Example of What We Learned: Equivalence

In Core-Plus the word "equivalent" is used two ways. *Equations* (and other statements) are equivalent if, as usual, each implies the other. *Expressions* are equivalent if they are two different forms for the same thing, that is, if they are equal for all values of the variables.

The first-named author was taken aback by the second use of equivalent. Why say that 2 and $1 + 1$ are equivalent, when we already have a better word: they are equal. Why say that $2x$ and $x + x$ are equivalent, when we already can say that $2x = x + x$ is an identity? Furthermore, he argued that Core-Plus, by defining equivalent for both equations and expressions, would run the danger of exacerbating the unfortunate tendency of students to think that equations and expressions are the same thing anyway.

Others (both writers and consultants) were taken aback that this author was taken aback! Although "equivalence" was not used for expressions in the algebra books this author had checked, some others had seen it and had also used it regularly. As the discussion went on (over many days before and after the 2004 meeting), it became clear that we were all trying to deal with the same issue, but simply had different ideas about what would work well. The issue is that many students don't have a good feel for expressions (or equations!). Rules for rewriting expressions are just formalisms for them, without much purpose. We wanted to make clear that different expressions for the same thing have different uses. For instance, $x^2 - 3x + 2$ and $(x - 1)(x - 2)$ are equivalent (ahem, setting them equal creates an identity), but the former is good for recognizing a quadratic whose graph opens up and the latter is good for identifying the zeros.

We thought about banishing the use of the word equivalent for expressions, but this has downsides too. One has to use longer and somewhat fuzzier phrases such as "different forms of the same thing" (but then what does "same thing" mean?) or "expressions which form an identity when set equal". Or we could just say "expressions that are equal", but this could cause confusion between equal expressions in an identity such as $2x + 4 = 2(x + 2)$, and situations where two expressions are set equal to form an equation such as $2x + 4 = 5$.

What did we learn? First, that what one mathematician assumes is standard is not always standard to another. Various members of the advisory group had rather different ideas about what was normal usage of "equivalent". Second, that what would appear to be a simple issue—just make a definition and be done with it—was not simple at all when it came to the ramifications for clarity in student minds. Either direction we took on equivalence had pitfalls for possible student confusion. Third, that people who had spent a lot of time in the precollege classroom had a lot of wisdom about what works well and what doesn't.

Conclusion

We have very much enjoyed our four years working with the Core-Plus developers. We look forward to the publication starting in 2007 of the second edition. We think we will have helped make the second edition another step forward; we know we have learned a lot in the process and have gained immense respect for the people who actually make it happen. We encourage any readers who are interested in precollege mathematics education to look for opportunities to be involved in projects that intrigue them.